# Fiscal non-linearities induced by an informative real-financial economic cycle*

André Casalis†

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**Abstract**

We investigate the combined effect of business and financial cycles on a non-linearly fluctuating economy, designing and estimating a joint economic cycle. A STVAR model and generalized impulse response analysis enable us to examine the non-linear effect on GDP of unanticipated government expenditure shocks, which we complement by performing scenario analysis to analyse how the model reacts during representative expansions and contractions of the economy. The main findings are that (i) every specification shows concordance between signs of shock and GDP response; (ii) the inclusion of an indicator of fiscal capacity in the model leaves the baseline key findings unchanged; (iii) the main results show diminishing returns to increasing expansionary stimuli; (iv) public debt and private credit generally behave pro-cyclically; (v) scenario analysis suggests higher yield to shocks during recessions.

Keywords: Markov Chain Monte Carlo; Nonlinear vector autoregression; Generalized Impulse Response Functions; Fiscal shock; Fiscal multiplier; Smooth transition; Business cycle; Financial cycle.
JEL Classification: C32; E17; E32; E37; E51; E62; H20; H63

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†University of York; e-mail: andre.casalis@gmail.com

# 1 Introduction

We consider a fluctuating economy along a combined economic cycle carrying information on both real activity and the financial sector, and focus on the state contingency of the effects of a fiscal stimulus. We contrast our results to a benchmark model estimated on a purely financial cycle and including fiscal and credit controls among the variables. Specifically, we adopt a non-linear Smooth Transition VAR and we show the cumulative effect of a government expenditure policy shock. We find that there are diminishing returns in terms of effect on GDP to a larger expansionary shock, an evidence which also appears in our contrast benchmark. Moreover, controlling for private credit and public debt does not qualitatively change the result shown by the baseline model. We attribute the stability of the key features of the specifications to the use of a cycle which already includes information about the financial sector and fiscal space. We also perform a scenario analysis exercise, delivering fiscal shocks either in an average contraction or in an average expansion of the cycle. We find unequivocal evidence that the fiscal expenditures multipliers are on average larger in a typical recession, rather that in an expansion, a result stable across all specifications used.

The cyclical behaviour of GDP – commonly known as the business cycle – has been widely accepted in the literature since Burns and Mitchell (1946) and, starting with Mankiw (1989), it has more recently been interpreted as the tell-tale sign of underlying economic fluctuations. Business cycle theories have by now become common and influential (Zarnowitz, 1992; Laidler, 1999; and Besomi, 2006). At the same time, even though the notion of financial booms and busts that could impact the economy is not new, the financial world came to assume an the ancillary role of either an accelerator or a delayer of the return to the natural steady state of the economy (Bernanke et al., 1999). Because of this, it came to be seen as something that could be ignored in first approximation (Woodford, 2003) and progressively disappeared from mainstream macroeconomics.

The financial crisis forcefully brought the spotlight back to the concept of "financially induced crisis" (Reinhart and Rogoff, 2014; Basel Committee on Banking Supervision, 2010; Jordà et al., 2017; and Ball, 2014) and triggered a growing advocacy to, in the words of Jordà et al. (2017), "take finance seriously". The intertwined nature of the real and financial economy has since been explored in depth. Arcand et al. (2015) consider whether there is a threshold over which the growth of the financial sector becomes detrimental to output. Credit and business cycle share a relationship investigated by Gertler and Kiyotaki (2015), while Ilzetzki et al. (2013) show how high levels of public debt make fiscal policy ineffective. The idea of a procyclicality of the financial system has become increasingly popular (Borio et al., 2001, Daníelsson et al., 2004, Kashyap and Stein, 2004, Brunnermeier et al., 2009, and Adrian and Song Shin, 2010), however there is still no broad consensus on what exactly a financial cycle is or how to measure it -with the notable exception of Drehmann et al. (2012).

The notion of time-varying behaviour within any given economy is crucial in the field of state contingency of fiscal multipliers, focusing on the ways in which the economy reacts differently to the same fiscal policy measure in different times. This amounts to believing that there exists a state variable on which fiscal multipliers are contingent, which in the literature is, commonly, considered to be the business cycle (Auerbach and Gorodnichenko, 2012; Callegari et al., 2012; Galvão and Owyang, 2018; Bolboaca and Fischer, 2019; Tenreyro and Thwaites, 2016; and Bruns and Piffer, 2019). We focus instead on a combined *economic cycle*, ideally merging the information coming from the business and the financial cycle, here loosely defined as the medium- and long-run fluctuations of various housing, interest rate, and stock market component variables. We make our economy proceed along such a cycle in an effort to better reproduce its evolution accounting for both real and financial drivers.

We specifically control for private credit to non-financial institutions and public debt, both normalized by GDP. The emphasis on measures of financial stress and fiscal burden comes from a deliberate effort to take the finance sector seriously. Gertler and Kiyotaki (2015) have already explored the pro-cyclical inter-linkages between credit and the business cycle, while the stifling effect of public debt on growth has already been substantiated in Reinhart and Rogoff (2011) and later further confirmed by Poghosyan (2018), whose findings – an asymmetrical relation between financial and debt cycles – particularly complement our own evidence of an asymmetrical and non-proportional output reaction to fiscal shocks. Moreover, we also find some empirical evidence that a growing amount of public debt is associated with a crippled GDP expansion, a crucial result already showcased in Ilzetzki et al. (2013) that highlights the complex relationship between public debt and economic growth.

The fluctuation along the economic cycle is reproduced using the approach of Auerbach and Gorodnichenko (2012, henceforth AG): a Smooth Transition VAR able to smoothly change the coefficients between two extreme regimes (a state of absolute contraction or expansion of the economy). The choice of a non-linear model is supported on one side by a growing awareness in the literature that complex phenomena require non-linear modelling techniques. [1]

Building further on the approach of AG, we focus on our economic cycle and we augment the model with, in turn, private credit and public debt. Furthermore, we add to the strictly linear impulse responses, as we use the generalized impulse response functions – GIRF – analysis pioneered by Koop et al. (1996). Detaching from the original approach, we are able to let go of the unintuitive assumption that after the shock is delivered, the model is stuck in one perpetual phase of the cycle, *de facto* suppressing the non-linear nature of the analysis. Generalised impulses are a powerful technique allowing enough flexibility to set the economy free to evolve according to its own mechanics. We further modify the original GIRF – the algorithm of Pesaran and Shin (1998) – to allow for structural government expenditure shocks.

The evidence drawn from GIRF analysis strongly supports the use of a cycle rich in information about the financial sector. Interestingly, we find that extending our baseline yields the same key features of the baseline specification, while a much more significant change in results is observed when we change the scenario in which the fiscal shock is delivered. From a policy perspective, the crucial result that expansionary stimuli are subject to diminishing returns questions the ability of fiscal policy to boost the economy at all. At the same time, it becomes evident that a better knowledge of the non-linear interactions taking place inside an economy is crucial to policy makers who want to make informed and efficient decisions.

The rest of this chapter is organized as follows. Section 2 details the model and the data, and Section 3 presents the empirical results. Section 4 concludes.

[1] A more compelling case for a change of perspective in economic modelling can be found in Chiu and Hacioglu Hoke (2016b) and Chiu and Hacioglu Hoke (2016a).

## 2 Methodology

This section presents the model and discusses generalized impulse response analysis that will be used to investigate its dynamics.

### 2.1 The Smooth Transition VAR model

Our model of choice, the STVAR, is the multivariate extension by van Dijk et al. (2002) of the univariate Smooth Transition AR introduced by Granger and Teräsvirta (1993). A further extension by AG adds the Smooth Transition dynamics to the variance-covariance matrix of the innovation process, allowing it to also become state-contingent. The econometric specification is as follows:

$$\mathbf{X}_t = [(1 - F(z_{t-1}))\mathbf{\Pi}_E + F(z_{t-1})\mathbf{\Pi}_C]\,(L)\mathbf{X}_{t-1} + \mathbf{u}_t \tag{1}$$

$$\mathbf{u}_t \sim N(\mathbf{0}, \mathbf{\Omega}_t) \tag{2}$$

$$\mathbf{\Omega}_t = \mathbf{\Omega}_E(1 - F(z_{t-1})) + \mathbf{\Omega}_C F(z_{t-1}) \tag{3}$$

$$F(z_t) = \frac{e^{-\gamma z_t}}{1 + e^{-\gamma z_t}} \qquad \gamma > 0$$

$$\mathrm{Var}(z) = 1 \qquad \mathrm{E}[z] = 0,$$

$\mathbf{\Pi}_E$ and $\mathbf{\Pi}_C$ are the coefficient matrices corresponding to the extreme states of the cycle and $\mathbf{X}$ is the data matrix. The transition function $0 \leq F \leq 1$ governing the shift between the phases is in turn determined by the state-contingent variable $z$. $\gamma$ is the parameter controlling the speed and the smoothness of the transition; the subscripts $E$ and $C$ again refer respectively to expansion and contraction phases of the cycle.
The model has two channels of transmission for shocks. The dynamic channel goes through the lag polynomials $\mathbf{\Pi}_E(L)$ and $\mathbf{\Pi}_C(L)$ in Equation (1), while the state-contingent variance-covariance matrix $\mathbf{\Omega}_t$ in equations (2)-(3) acts as a contemporaneous propagation mechanism. The model features a large number of parameters to be estimated and it shows true non-linearity in the parameters, since the data matrix will be augmented with the economic cycle. However, after taking the first order condition as in Equation 4, it becomes apparent that the model becomes linear for any given guess of the variance-covariance matrices s$\mathbf{\Omega}_E$ and $\mathbf{\Omega}_C$ and the computation of the coefficient matrix $\mathbf{\Pi} = [\mathbf{\Pi}_E, \mathbf{\Pi}_R]$ becomes trivial.

$$\begin{aligned}&\mathrm{Vec}\left[\mathbf{\Pi}'\right] = \left(\sum_{t=1}^{T}\left[\mathbf{\Omega}_t^{-1} \otimes \mathbf{W}_t'\mathbf{W}_t\right]\right)^{-1} \mathrm{Vec}\left[\sum_{t=1}^{T}\mathbf{W}_t'\mathbf{X}_t\mathbf{\Omega}_t^{-1}\right]\\ &\text{Where}\\ &\mathbf{W}_t = [(1 - F(z_{t-1}))\mathbf{X}_{t-1},\ F(z_{t-1})\mathbf{X}_{t-1}\ldots\ (1 - F(z_{t-1}))\mathbf{X}_{t-p},\ F(z_{t-1})\mathbf{X}_{t-p}]\end{aligned} \tag{4}$$

We apply the same estimation strategy as AG, as described in Appendix A, and we use the Markov Chain Monte Carlo method presented in Chernozhukov and Hong (2003), with Metropolis-Hastings algorithm and flat priors, to build building up a sequence of guesses leading to the highest likelihood. While the overall estimation procedure has Bayesian features, the model estimation step sees the use of GLS.

## 2.2 Impulse response functions

Since the model we are going to use possesses interesting non-linear features, we would like to preserve them in the analysis phase. This is not a trivial endeavour: the original AG work featured linear orthogonalized impulse response functions, assuming that the the model would perpetually stay in the same phase in which the shock was delivered. We regard such an assumption as generally difficult to defend and contrasting with the whole spirit of this investigation. Therefore, we turn to the generalised impulse response analysis pioneered by Koop et al. (1996) and further described by Pesaran and Shin (1998).

The intuitive definition of the future effect of a shock on a system is the difference between the expectation of the shocked system and that of a baseline where the shock never happened. The formal definition of generalized impulse is as follows

$$\begin{aligned} GI_{\mathbf{X}}(h, s_t, \mathcal{H}_{t-1}) = \mathrm{E}\left[\mathbf{X}_{t+h} \mid \mathcal{H}_{t-1}, s_t\right] - \mathrm{E}\left[\mathbf{X}_{t+h} \mid \mathcal{H}_{t-1}\right], \\ \text{for the horizon } h = 0, 1, \ldots . \end{aligned} \tag{5}$$

The generalised impulse $GI_{\mathbf{X}}(h, s_t, \mathcal{H}_{t-1})$ is defined as the difference between the system expectation conditional on the history of realizations ($\mathcal{H}$) or on the history *and* the shock ($s$), thus averaging out future innovations that do not interest us. Both the conditional expectations can be seen as random variables, which makes $GI$ a random variable itself. Since our model is known and specified, we can compute the expectations and then estimate the empirical distribution of $GI$. It is then sufficient to pick a measure of centrality of the distribution as the estimate of the shock and one of dispersion to serve as error.

We further develop the traditional analysis and we slightly modify the algorithm, as suggested by Kilian and Vigfusson (2011) and Pellegrino (2021). We use the model reduced-form residuals to estimate the structural innovations, therefore identifying fiscal shocks, via the usual short-run recursive restriction of the Cholesky decomposition. The algorithm modification sacrifices the traditional irrelevance of the ordering of variables, one of the distinctive features of the traditional GIRF approach.

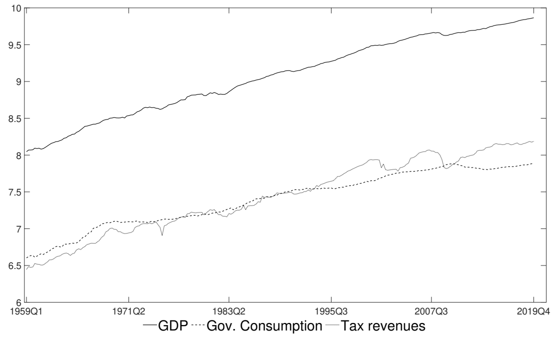
# 3 Empirical analysis

In what follows, we detail the variables and the data included in the analysis, and discuss the estimation strategy followed to create our economic cycle. A selection of empirical results is also presented.

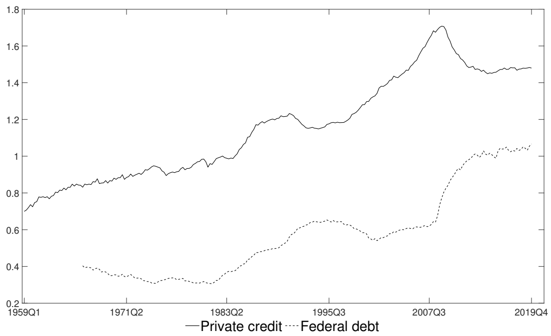
## 3.1 Variables and data

We use U.S. quarterly data from 1966Q1 (1952Q2, for the specification not including debt) to 2019Q4. Figure 1 presents our variables: Government expenditure, tax receipts and GDP are all log real series; public debt and credit to private non financial institutions (for short, henceforth private credit) are normalized by GDP.

Figure 1: The data: macroeconomic and financial variables




(a) Macroeconomic variables




(b) Financial variables

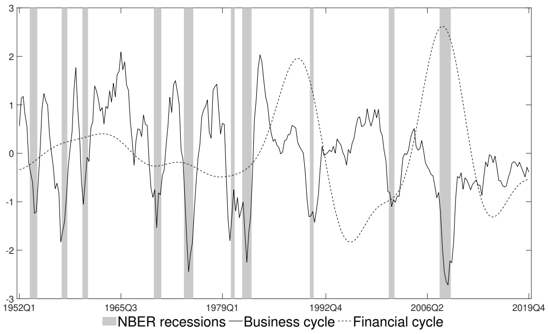
Source: Bureau of Economic Analysis.
Note: Log real data of (a) Government Expenditure, Tax Revenues, GDP, and (b) Public Debt, Private Credit (both normalized by GDP).

Government expenditure, tax revenues, and GDP has constituted the standard selection when estimating fiscal multipliers ever since Blanchard and Perotti (2002). Every specification includes the estimate of the economic cycle to allow for dynamic computation of truly non-linear impulse responses. The choice of private credit as a single indicator of financial stress is justified by the findings of Borio (2014) and Drehmann et al. (2012), who explicitly single out this variable as the carrier of all information on the financial sector. We specifically choose public debt in light of its relationship with fiscal multipliers identified by Perotti (1999) and Ilzetzki et al. (2013), both finding that high levels of fiscal burden (debt-to-gdp ratio) are able to impair fiscal policy, shrinking the size of fiscal multipliers. Furthermore, we believe that model already features implicit deficit dynamics (as it includes public expenditures and revenues) and we see the inclusion of public debt as a natural complement. We estimate the model in first differences to ensure stationarity.

## 3.2 The financial cycle

To obtain an estimate of the financial cycle used to contrast and benchmark the main results, we adopt the approach of Drehmann et al. (2012) and Borio (2014), relying on frequency analysis. Specifically, we apply the Christiano and Fitzgerald (2003) passband filter to isolate and extract the so called medium-term frequency components of the cycle, that is the components oscillating with a frequency between 32 and 120 quarters (8 and 30 years). The choice of frequency analysis over the longer historied turning-point analysis is dictated by the need to have an explicit value of the cycle for each quarter, instead of an estimate of maximum and minimum points.
Our result is comparable with previous literature estimates, even if we drastically reduce the number of variables considered from five to just one: the credit to private non-financial institutions, normalized by GDP. This choice allows us to base the estimation of the financial cycle on one of the variables included in the model specification, as required for the use of generalized impulse response analysis. To allow a comparison, we also include an estimate of the business cycle obtained as in AG. Figure 2 below shows our estimate of the financial cycle. NBER recessions and a simple estimate of the business cycle are also reported for comparison.

Figure 2: Business cycle, financial cycle and NBER chronology




Note: The business cycle is the $MA(7)$ of the ouput growth; the financial cycle is obtained via a band pass filter extracting the components fluctuating with frequency 32-120 quarters.

### 3.3 The economic cycle

The leading intuition behind our economic cycle is that the economy is contemporaneously under the influence of both a real and a financial cycle. To retrieve a measure able to include information about both worlds, we make use of FRED-MD, a macroeconomic database of 128 variables related to the U.S. economy at monthly frequency. The database, an ideal extension of the work of Stock and Watson (1996), is described and detailed at length in the accompanying paper (McCracken and Ng, 2016), and partially in Appendix C. The panel is formed by 742 monthly observations, from 1959:01 to 2020:10, but we limit the data we use to 2019:12, in line with the macro and financial variables. The first 2 observations are lost to perform data transformation to achieve stationarity and several series have missing observations at the beginning of the sample, making the panel unbalanced.

To build a synthetic measure of a comprehensive real and financial cycle, we follow a three-step approach. First, we need to efficiently extract the information from the monthly series. To this purpose, we follow McCracken and Ng (2016) in reducing the dimension of our database with a factor analysis strategy. Second, using the same reasoning we used estimating the Financial cycle, we filter the factor scores to isolate the short- and medium-term components of the business cycle and the medium- and long-run frequencies of the financial cycle. Third, we project GDP onto the components to estimate the overall economic cycle and smooth it with a year long moving average.

It is well established that in large $T$ and $N$ settings,[2] static or dynamic principal components can be a consistent estimate of latent factors (see, Forni et al. (2005, 2000); Stock and Watson (2006); Boivin and Ng (2005); Bai and Ng (2008)). Since we have missing observations, we estimate the factors using the EM algorithm given in Stock and Watson (2002), which allows for a conveniently simple treatment of missing observations. After demeaning and standardizing the series, in the first iteration of the algorithm we rebalance the panel, initializing all empty observations to 0. Given a number $r$ of factors, we estimate matrix $T \times r$ of factor scores $\mathbf{F} = (f_1, \ldots, f_T)$ paired with a $N \times r$ matrix of loadings $\mathbf{\Lambda} = (\lambda_1, \ldots, \lambda_N)'$ under the normalization $\frac{\mathbf{\Lambda}'\mathbf{\Lambda}}{N} = \mathbf{I}_r$. For each missing observation $t$ of the $i$th series, the initial 0 guess is updated to $\widehat{\lambda_i}' \widehat{f_t}$, multiplied by the standard deviation of the series. Finally, the mean is added back and the resulting value is considered the $t$th observations of the $i$th series, which we demean and

[2]Where $T$ and $N$ are, respectively, the number of observations and the number of variables.

standardize again with the updated mean and standard deviation. The algorithm iterates until the estimated factors do not change any more.

Several criteria, imposing different assumptions upon the factor model, are available to find the optimal $r$ number of significant factors. Bai and Ng (2002) proposed the $PC_p$ criteria, which minimize the number of factors chosen, imposing a penalty of $\frac{\log(\min(N,T))}{\min(N,T)}$ to keep the model parsimonious. Since $\min(N,T)^{-1} \approx \frac{N+T}{NT}$ when $N,T \to \infty$, several functional forms of the criteria can be specified. We choose the specification with the better finite sample properties, $\frac{N+T}{T}\log(\min(N,T))$, corresponding to the $PC_{p2}$ criterion in Bai and Ng (2002). The criterion selects seven significant factors, eight if the sample is not limited to 2019, as Appendix D shows.

Once the $r = 7$ factors are estimated, we regress each series on an increasing subset of them to compute a measure of how much variability the orthogonal factors are able to explain for each series. That is, for the $i$th series and for each factor $k = 1, \ldots, r$ we compute $R_i^2(k)$ and an average across series yields how much a $k$ given number of factors explain of our panel: $R^2(k) = \frac{1}{N}\sum_{i=1}^{N} R_i^2(k)$. Similarly, the marginal gain in explanatory power for the $i$th series obtained from adding an extra factor is, from the second factor onward, the difference in the $i$th series R-square values $mR_i^2(k) = R_i^2(k) - R_i^2(k-1),\ k = 2, \ldots, r$. In the case of the single-factor subset, the additional explanatory power trivially coincides with the overall variance explained, so that $mR_i^2(1) = R_i^2(1)$. We can compute how much adding a factor on average increases the average explanatory power over the whole panel, taking the mean of the marginal gains across the series $mR^2(k) = \frac{1}{N}\sum_{i=1}^{N} mR_i^2(k)$.

Table 1 lists the overall variance explained by the factors, $R^2(r)$, along with the ten series which load the most on each $k$th factor; that is, the series featuring the highest $mR_i^2(k)$. A description of all the variables used in the analysis is available in Appendix C.

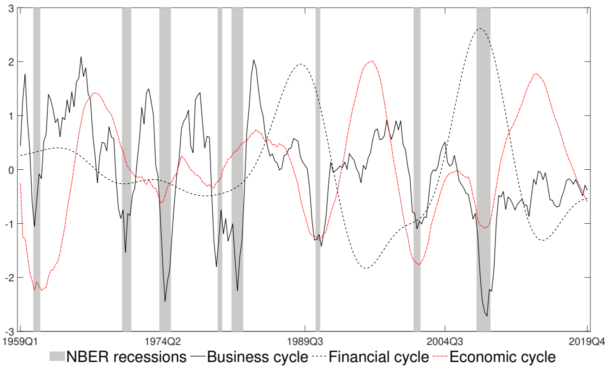
Table 1: Estimated factors and heavy loading series - $R^2(7) = 0.4480$

| $mR^2(1)$ | 0.1441 | $mR^2(2)$ | 0.0718 | $mR^2(3)$ | 0.0680 | $mR^2(4)$ | 0.0551 |
|---|---|---|---|---|---|---|---|
| payems | 0.7092 | cusr0000sac | 0.6921 | aaaffm | 0.5399 | gs1 | 0.5123 |
| usgood | 0.7006 | dndgrg3m086sbea | 0.6805 | t10yffm | 0.5315 | gs5 | 0.5004 |
| ipmansics | 0.6833 | cusr0000sa0l2 | 0.6593 | baaffm | 0.5163 | aaa | 0.4894 |
| indpro | 0.6513 | cpiaucsl | 0.6407 | t5yffm | 0.4746 | tb6ms | 0.4714 |
| manemp | 0.6430 | cusr0000sa0l5 | 0.6053 | tb3smffm | 0.4108 | gs10 | 0.4602 |
| dmanemp | 0.6112 | cpitrnsl | 0.5816 | tb6smffm | 0.3926 | baa | 0.4488 |
| ipfpnss | 0.6034 | pcepi | 0.5783 | t1yffm | 0.3318 | cp3mx | 0.3757 |
| cumfns | 0.5897 | cpiulfsl | 0.5232 | houst | 0.2364 | tb3ms | 0.3732 |
| ipfinal | 0.5041 | wpsfd49502 | 0.4593 | houstmw | 0.1954 | twexafegsmthx | 0.2230 |
| ipdmat | 0.4751 | wpsfd49207 | 0.4417 | houstne | 0.1910 | s&p div yield | 0.1975 |
| $mR^2(5)$ | 0.0431 | $mR^2(6)$ | 0.0342 | $mR^2(7)$ | 0.0317 | | |
| t1yffm | 0.3174 | awhman | 0.2695 | s&p 500 | 0.4945 | | |
| tb6smffm | 0.2705 | ces0600000007 | 0.2632 | s&p: indust | 0.4915 | | |
| t5yffm | 0.2443 | uemp15ov | 0.2045 | s&p div yield | 0.3655 | | |
| tb3smffm | 0.2288 | s&p pe ratio | 0.1807 | s&p pe ratio | 0.2560 | | |
| permit | 0.2231 | uemp27ov | 0.1668 | umcsentx | 0.2341 | | |
| permitw | 0.2147 | acogno | 0.1478 | vxoclsx | 0.1836 | | |
| houstw | 0.1985 | isratiox | 0.1464 | ipcongd | 0.0864 | | |
| t10yffm | 0.1869 | ipcongd | 0.1400 | excausx | 0.0640 | | |
| houst | 0.1753 | s&p div yield | 0.1174 | ipfinal | 0.0621 | | |
| compapffx | 0.1740 | uempmean | 0.1007 | ipdcongd | 0.0501 | | |

Note: Seven factors selected by the $PC_{p2}$ criterion and the ten series loading the most on each factor. The table also reports the total variation explained by the seven factors ($R^2(7)$), and the additional variation explained by adding the $k$th factor ($mR^2(k)$). As an example, the seven factors explain together 44.80% of the panel variation, while $mR^2(1) = 0.1441$ is the quota explained solely by the first factor. Moreover, 0.7092 is the fraction of variation in the variable payems explained by the first factor.

Factor 1 explains 0.1441 of the variation in the data and is easily interpreted as a real activity factor, since it is mostly loaded by series relating to industrial production and employment. The second factor, contributing 0.0718 to the whole variation in data, mainly affects price variables and can be read as an inflation factor. Both the third and the fifth factors feature forward-looking variables such as term interest rates spreads and inventories, with a more modest contribution from real estate variables. Factor 4 is dominated by interest rate variables, and factor 6 contributes mostly to employment variables, with some influence from stock market and financial variables. The last factor mostly explains stock market variables. Figure 3 shows the end product of our strategy, an economic cycle built from information on both the real economy and financial variables, also including employment and stock market information. A simple estimate of business cycle and the financial cycle which is central for the benchmark analysis are included for comparison. The economic cycle appears broadly well correlated with the NBER recessionary periods, while at the same time featuring a smoothness and an amplitude closer to the financial oscillations, rather than to the business cycle.

Figure 3: The economic cycle




Note: The business cycle is the $MA(7)$ of the ouput growth; the financial cycle is obtained via a band pass filter extracting the components fluctuating with frequency 32-120 quarters. The economic cycle is the smoothed GDP projection onto seven factor scores carrying information about the real economy, production, interest rates and financial markets.

### 3.4 Impulse responses

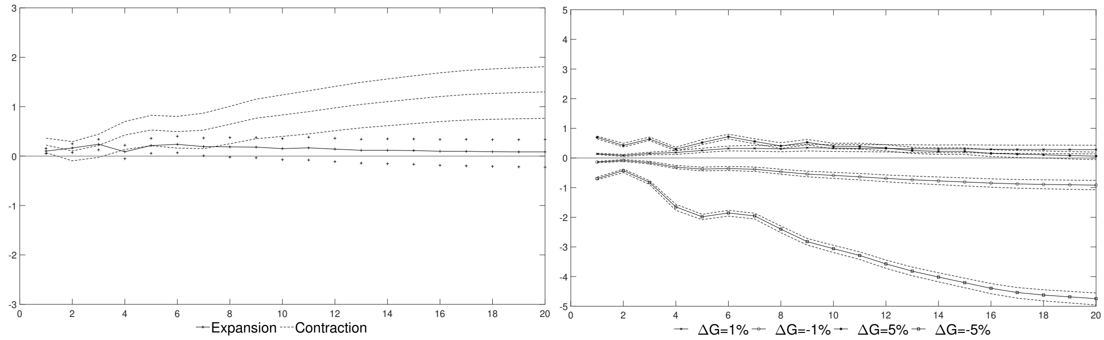
Our focus is on the response of GDP to a fiscal government expenditure shock. We consider three different specifications of the variables in $\mathbf{X}_t$: a standard BP-like $\mathbf{X}_t = [z_t,\ g_t,\ \tau_t,\ y_t]$, acting as the baseline, and two extended specifications, one with public debt and the other with private credit, in line with the approach we took for the benchmark baseline —while sample limitations prevent us from adopting a specification augmented with both. Let $g$ denote government expenditure; $\tau$ is tax revenues; $y$ is GDP; $Pc$ denotes private credit (normalized by GDP); and $d$ denotes public debt (normalized by GDP). All variables are first differences of the log real series and each STVAR model is augmented by the estimate of the economic cycle, denoted by the variable $z$. The shocks considered roughly correspond to $\pm 0.15\%$ and $\pm 0.8\%$ of GDP, namely $\pm 1\%$ and $\pm 5\%$ of U.S. government expenditure. The choice of a 5% shock is in line with the American Recovery and Reinvestment Act (ARRA) of 2009 (2009) stimulus package, which delivered an estimated combined impact of roughly 2.5% of GDP in the first year of enactment, as detailed in The Congress of the United States - Congressional Budget Office (2012). Furthermore, the most recent recession is already calling for an extremely large stimulus package, rumoured to be around 10% of GDP in total size.

We perform a scenario analysis, delivering a shock when the model is artificially brought to an average, representative recession or expansion. This allows us to investigate the consequences of shocks of several sizes and signs impacting an economy in a well-defined state. We use our economic cycle to discriminate strong expansions from deep recessions and we select the quarters for each regime together with their lags, that is we build two synthetic histories of realizations. After taking the median, we augment our natural history with this synthetic data, effectively feeding the autoregressive mechanism of the model with representative values of the regime of interest, simulating a state of the economy to be in a median recession or expansion.

#### 3.4.1 Baseline specification

We first present results for the baseline specification $\mathbf{X}_t = [z_t,\ g_t,\ \tau_t,\ y_t]$, which is consistent with the model specification used by Blanchard and Perotti (2002). Figure 4 shows both the linear and the non-linear impulse responses. Linear IRFs are used as a comparison in the unlikely scenario in which the transition function is stuck to either 0 or 1, thus collapsing the model to a standard linear VAR.

Figure 4: Baseline specification, GDP reaction




(a) Linear IRF (b) Generalised IRF

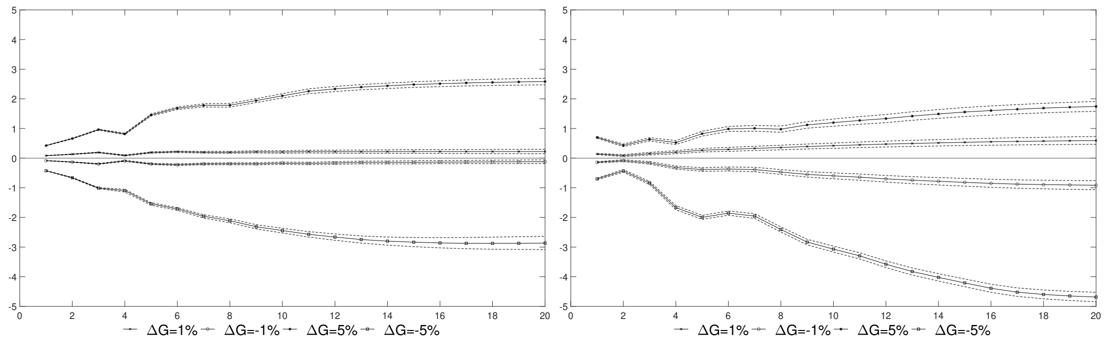
Note: Cumulative linear (a) and generalized (b) impulse responses. Percentage GDP response to a unit standard deviation (a) or to percentages of government expenditure (b) fiscal shock. $\Delta G$ denotes the variation in government expenditure, the percentage is the size of the shock. STVAR includes public expenditure, tax revenues, and GDP. Confidence bands are at 5th and 95th percentile.

Overall, the results appear similar to the debt-augmented benchmarke specification we show in Appendix B.1.3, with an even clearer message. Linear responses appear rather stable after the very short horizon. Furthermore, they point unequivocally to fiscal shock in recession being more effective than in expansion, especially considering the longer horizon, a result we will investigate further with the scenario analysis exercise. The generalized responses, on the other hand, present many of the points we already made in the previous chapter. There is concordance between the sign of the shock and the sign of the response, since a positive fiscal stimulus will have a positive effect on GDP and a budget cut will, on the other hand, yield a contractionary effect. We find a clear phenomenon of diminishing returns to increasing expansionary stimuli, to the point where a larger budget expansion will only lead to a larger effect in the short-medium horizon. A larger cut in expenditure, on the other hand, appears to cause a smooth decline in GDP, taking a longer time to stabilize. Furthermore, contrary to the benchmark there is no difference between the impact and the medium-long horizon of the responses, with the effect being consistently positive or negative for all the quarters.

### 3.4.2 Scenario analysis, baseline specification

To shed some light on the question of whether expenditure multipliers are larger in (a typical) recession or expansion, Figure 5 presents evidence from the scenario analysis exercise using only the baseline specification.

Figure 5: Baseline specification, scenario analysis generalized IRFs




(a) Typical expansion (b) Typical recession

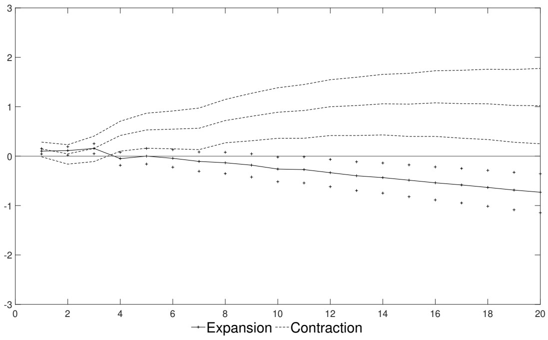
Note: Cumulative generalized impulse responses to a fiscal shock delivered in a median representative recession or expansion. $\Delta G$ denotes the variation in government expenditure, the percentage is the size of the shock. STVAR includes public expenditure, tax revenues, and GDP. Confidence bands are at 5th and 95th percentile.

A number of differences appear evident as we compare the two scenarios. In a typical expansion, the GDP reaction to a fiscal shock appears to be more symmetric to the sign of the shock. Even the phenomenon of diminishing returns is weak to the point of being negligible. A representative recession yields a more diverse reaction to a fiscal expenditure shock. The responses are no longer symmetric and negative shocks produce a larger effect. Moreover, it is again evident that there are diminishing returns to larger expansionary measures. At the same time, some key characteristics noted in the baseline scenario still hold in this exercise, such as the concordance in sign between impact and long-run response. Crucially, positive shocks are still expansionary, and budget cuts lead to recession in both scenarios. Overall, whether fiscal multipliers are larger during a recession rather than during an expansion depends on the size and on the sign of the shock. The effect on GDP of a small shock, irrespective of the sign, appears to be larger in absolute value during a typical recession. However, non-linearities kick in when the size of the budget increase is scaled up, causing a large fiscal stimulus to yield less effect on GDP during a period of crisis than with a prospering economy. The results clearly suggest that the classical notion of larger multipliers in a recession needs to be revised to carefully account for the size and sign of the shock.

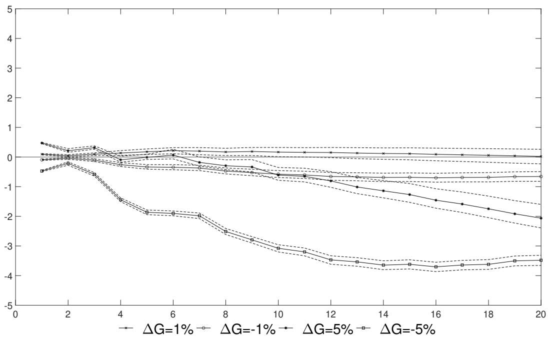
### 3.4.3 Augmented specifications

Next we show our findings when the model specification is changed to include either private credit or public debt, both normalized by GDP. Figure 6 presents linear and non-linear GDP impulses responses for these extended specifications.

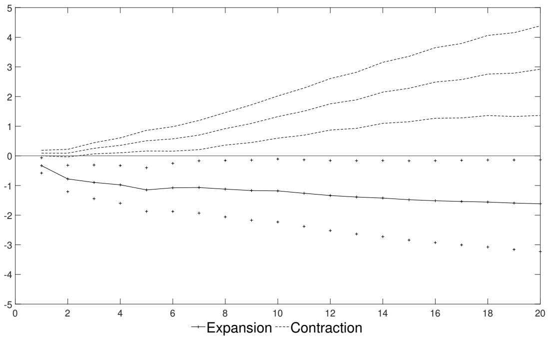
Figure 6: Augmented specifications with private credit or public debt, GDP reaction



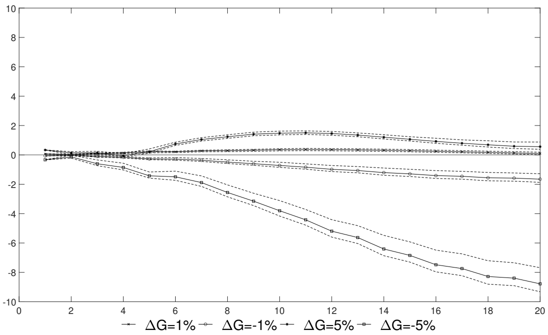

(a) Linear IRF - Private credit




(b) Generalised IRF - Private credit




(c) Linear IRF - Public debt

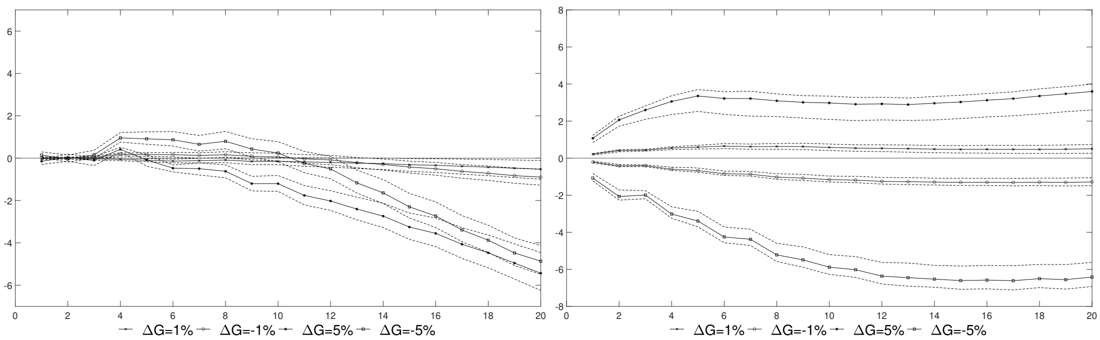



(d) Generalised IRF - Public debt

Note: Cumulative linear (a, c) and generalised (b, d) impulse responses. Percentage GDP response to a unit standard deviation (a, c) or to percentages of government expenditure (b, d) fiscal shock. $\Delta G$ denotes the variation in government expenditure, the percentage is the size of the shock. STVAR includes public expenditure, tax revenues, GDP, and either private credit or public debt. Confidence bands are at 5th and 95th percentile.

The interpretation of the extended specifications, read together with the behaviour of the extension variables presented in Figure 7, is broadly in line with the baseline key findings. In both the extensions, positive shocks still yield positive GDP effects and vice versa. Furthermore, the diminishing return of expansionary stimuli is still in place. A notable exception is the GDP reaction following a large expenditure increase in the case in which we check for private credit. This pairs with the behaviour of the credit-to-GDP ratio itself, which is similar in the case of large shocks, regardless of their sign. The evidence suggests that the credit mechanism is also responsive to the size of the shock, and that a large expansionary shock may trigger a negative feedback on the economy. The debt-to-GDP ratio also appears strongly procyclical, in line with what observed in the benchmark. Furthermore, it can observed that the reaction of the debt-to-GDP ratio to a larger shock is proportionally larger than that of GDP and that the growth in the initial quarters of the response is faster. An interpretation in line with Perotti (1999) and Ilzetzki et al. (2013) is that the outstanding debt stock increase ends up impairing a further GDP expansion.

Figure 7: Augmented specifications, generalised IRFs for the augmentation variables



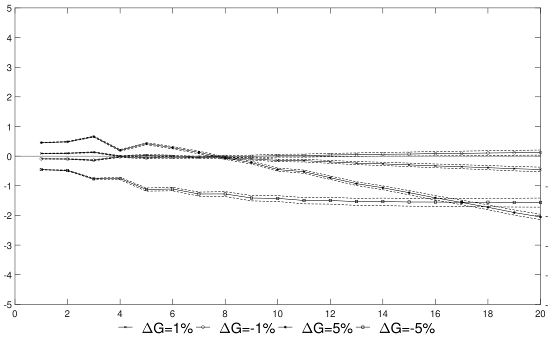

(a) Credit-to-GDP (b) Debt-to-GDP

Note: Cumulative generalised impulse responses. Percentage private credit or public debt response to a fiscal shock. $\Delta G$ denotes the variation in government expenditure, the percentage is the size of the shock. STVAR includes an estimate of combined cycle, government expenditure, tax revenues, GDP, and either public debt or private credit. Confidence bands are at 5th and 95th percentile.

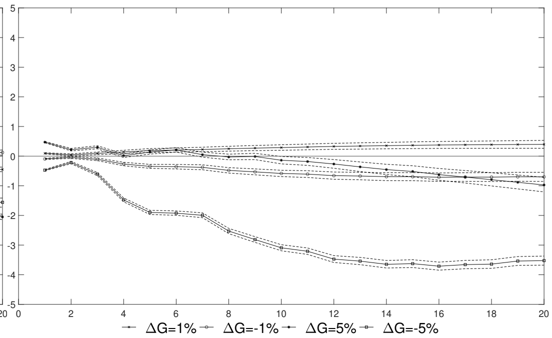
### 3.4.4 Scenario analysis, augmented specifications

Figure 8 presents the scenario analysis exercise for both the extended specifications. All the responses are well defined, with the exception of the larger shocks in the typical expansion scenario for the debt-extended specification, which are reported only in Appendix F.1 for better readability of results. The overall conclusion is consistent with the findings brought forward in the benchmark specification and the empirical evidence presented for the baseline specification in this chapter in Section 3.4.2.

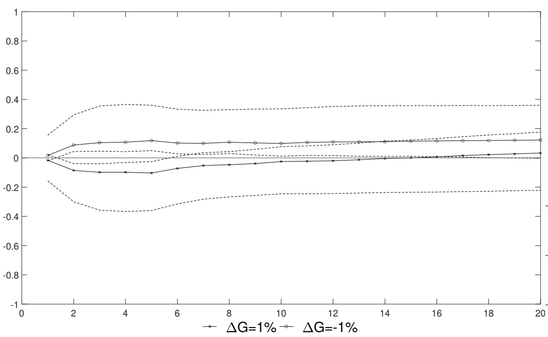
Figure 8: Augmented specifications, GIRFs for scenario analysis



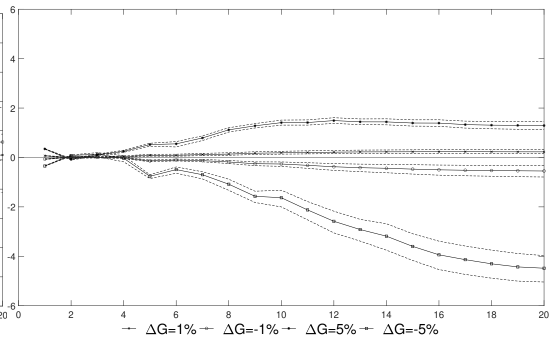

(a) Credit, typical expansion




(b) Credit, typical recession




(c) Debt, typical expansion

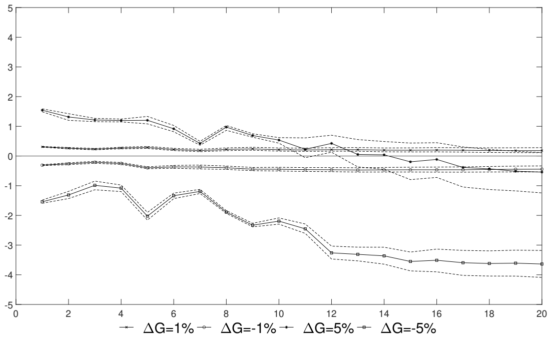



(d) Debt, typical recession

Note: Cumulative percentage GDP response to a fiscal shock. $\Delta G$ denotes the variation in government expenditure, the percentage is the size of the shock. The STVAR includes an estimate of combined cycle, government expenditure, tax revenues, GDP and either private credit (a and b) or public debt (c and d). Confidence bands are at 5th and 95th percentile.

Figure 9 shows a simple extension to the scenario analysis exercise, limited to the specification augmented with public debt-to-GDP. We assume that the entirety of the fiscal shock translates onto debt via deficit, explicitly modelling a budget increase financed via debt and a debt reduction through a reduction in public expenditures. The model struggles to narrowly identify the dynamics yielded by fiscal shocks during typical expansions, which are reported only in Appendix F.1. In any case, a meaningful comparison can be reached, since the GDP dynamics in typical recessions fall outside the confidence bands for their typical expansion counterparts. Overall, the broad conclusion is again that GDP reaction to a fiscal expenditure shock is on average larger in absolute value during typical recessions.

Figure 9: Debt augmented specification, GIRFs for scenario analysis with a shock to public debt




Note: Cumulative generalised impulse responses to a fiscal shock delivered in a median representative recession. The same shock is applied with opposite signs to government expenditure and public debt. $\Delta G$ denotes the variation in government expenditure, the percentage is the size of the shock. The STVAR includes an estimate of combined cycle, government expenditure, tax revenues, public debt, and GDP. Confidence bands are at 5th and 95th percentile.

# 4 Conclusions and remarks

We estimated a combined economic cycle, a synthetic measure carrying information on both the real economy and production, and the financial cycle. Such an index appears well correlated with the official recession chronology published by the NBER and appears to have inherited the smoothness and the amplitude of movement of the financial cycle. Its features enable us to investigate the state-contingent response to a fiscal shock in a complex economy, where the real as well as the financial sectors exert their own measure of influence. We confirmed that an economy contingent on the economic cycle keeps some features induced by both the cycles, and even extending a baseline parsimonious specification does not change its key properties. Some overall crucial findings can be adduced among the abundant results yielded by our empirical investigations. The use of the economic cycle makes even the baseline specification assume the features of what in the benchmark case was an extended specification with measures of financial stress and fiscal burden.

We used a Smooth Transition VAR to allow the economy to fluctuate along the cycle and analysed the reaction of GDP to shocks of different sign and size. As we observed with more complete specifications in the previous chapter, asymmetries in terms of sign and size of the shock do emerge, mostly for larger expansionary shocks. Such asymmetries are brought to light by the non-linear features of the model and of the impulse responses, whereas in the same context a linear setting would suppress this richness of reaction, forcing symmetry in the results. The baseline specification shows unequivocal concordance between the sign of the shock and the sign of the response. These dynamics carry over to our extended specifications and scenario analysis exercises, in concordance with what an extended specification would yield when made contingent to a purely financial cycle, as seen in Appendix B. The most interesting empirical finding remains the phenomenon of limited returns to increasing expansionary stimuli or, in other words, the persistence of the evidence that it appears easier to tank an economy rather than to boost it, making the cost of a mistaken policy dangerously steep. From a policy perspective, these results further depart from the notion of expansionary budget cuts á la Alesina and Ardagna (2013). Furthermore, they appear consistent across specifications, and

across recession, as well as expansion, scenario. An analysis of the behaviour of public debt and private credit after the shock provides further insight: both private credit and public debt appear to be strongly pro-cyclical, broadly in line with what has already been established by previous literature on the relationship between sovereign debt and GDP.
The scenario analysis complements the results yielded by our specifications and further endorses the hypothesis of a larger multiplier during a recession, tempered by the diminishing returns of larger expansionary packages. Overall, the results seems to hint at the existence of an optimally sized measure, able to achieve the optimal efficiency between cost and result of the intervention. Overall, our results advocate caution in the context of the traditional countercyclical public intervention during recessions: the existence of a limiting mechanism to the effect of expansionary packages may result in a waste of public resources. On the other hand, the confirmation of the existence of complex dynamics between fiscal space, financial stress, and the economy as a whole calls for further investigation in this line of research, in order to unravel the structural interactions of such a complex relationship.

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# Appendix

## A Estimation procedure

The STVAR model is defined as

$$\mathbf{X}_t = \left[(1 - F(z_{t-1}))\mathbf{\Pi}_E + F(z_{t-1})\mathbf{\Pi}_R\right](L)\mathbf{X}_{t-1} + \mathbf{u}_t \tag{6}$$

$$\mathbf{u}_t \sim N(\mathbf{0}, \mathbf{\Omega}_t) \tag{7}$$

$$\mathbf{\Omega}_t = \mathbf{\Omega}_E(1 - F(z_{t-1})) + \mathbf{\Omega}_R F(z_{t-1}) \tag{8}$$

$$F(z_t) = \frac{e^{-\gamma z_t}}{1 + e^{-\gamma z_t}} \qquad \gamma > 0 \tag{9}$$

$$\mathrm{Var}(z) = 1 \qquad \mathrm{E}[z] = 0, \tag{10}$$

where $\mathbf{X}$ is the data matrix, $\mathbf{\Pi}_E$ and $\mathbf{\Pi}_C$ are the coefficient matrices; $z$ is the switching variable, ruling the transition on the cycle, and computed as the 7-quarters moving average of the GDP growth; and $0 \leq F \leq 1$ is the smoothing function. The subscripts $E$ and $R$ refer respectively to expansion and recession phases of the business cycle.
The model log-likelihood is given by

$$\mathcal{L} = a - \frac{1}{2}\sum_{t=1}^{T}\log\left(|\mathbf{\Omega}_t|\right) - \frac{1}{2}\sum_{t=1}^{T}\mathbf{u}_t'\mathbf{\Omega}_t^{-1}\mathbf{u}_t \tag{11}$$

where

$$u_t = \mathbf{X}_t - (1 - F(z_{t-1}))\mathbf{\Pi}_E(L)\mathbf{X}_{t-1} - F(z_{t-1})\mathbf{\Pi}_R(L)\mathbf{X}_{t-1} \tag{12}$$

and $a$ is a constant.
The model has many parameters $\mathbf{\Psi} = \{\gamma,\ \mathbf{\Omega}_E,\ \mathbf{\Omega}_R,\ \mathbf{\Pi}_E,\ \mathbf{\Pi}_R\}$ and, as it appears from Equation 11, becomes linear in the lag polynomials $\{\mathbf{\Pi}_E,\ \mathbf{\Pi}_R\}$ for any guess of $\{\gamma,\ \mathbf{\Omega}_E,\ \mathbf{\Omega}_R\}$. The lag polynomials can be estimated with weighted least squares, where the weights are given by $\mathbf{\Omega}_t^{-1}$ and the estimates must minimize the target function

$$\frac{1}{2}\sum_{t=1}^{T}\mathbf{u}_t'\mathbf{\Omega}_t^{-1}\mathbf{u}_t \tag{13}$$

We set $\mathbf{\Pi} = [\mathbf{\Pi}_E, \mathbf{\Pi}_R]$ and then build an extended vector of regressors

$$\mathbf{W}_t = \left[(1 - F(z_{t-1}))\mathbf{X}_{t-1},\ F(z_{t-1})\mathbf{X}_{t-1} \ldots\ (1 - F(z_{t-1}))\mathbf{X}_{t-p},\ F(z_{t-1})\mathbf{X}_{t-p}\right] \tag{14}$$

so that we can rewrite Equation 12 in a more compact form, $\mathbf{u}_t = \mathbf{X}_t - \mathbf{\Pi}\mathbf{W}_t'$. The target function 13 can then be rewritten as

$$\frac{1}{2}\sum_{t=1}^{T}\left(\mathbf{X}_t - \mathbf{\Pi}\mathbf{W}_t'\right)'\mathbf{\Omega}_t^{-1}\left(\mathbf{X}_t - \mathbf{\Pi}\mathbf{W}_t'\right) \tag{15}$$

Taking the first order condition with respect to $\mathbf{\Pi}$:

$$\sum_{t=1}^{T}(\mathbf{W}_t'\mathbf{X}_t\mathbf{\Omega}_t^{-1} - \mathbf{W}_t'\mathbf{W}_t\mathbf{\Pi}'\mathbf{\Omega}_t^{-1}) = 0 \tag{16}$$

we rewrite it as

$$\sum_{t=1}^{T}\mathbf{W}_t'\mathbf{X}_t\mathbf{\Omega}_t^{-1} = \sum_{t=1}^{T}\mathbf{W}_t'\mathbf{W}_t\mathbf{\Pi}_t'\mathbf{\Omega}_t^{-1} \tag{17}$$

and apply the vectorization operator

$$\text{Vec}\left[\sum_{t=1}^{T}\mathbf{W}_t'\mathbf{X}_t\mathbf{\Omega}_t^{-1}\right] = \sum_{t=1}^{T}\text{Vec}\left[\mathbf{W}_t'\mathbf{W}_t\mathbf{\Pi}_t'\mathbf{\Omega}_t^{-1}\right] \tag{18}$$

Applying the properties of the Kronecker operator, the equation becomes

$$= \sum_{t=1}^{T}\text{Vec}\left[\mathbf{\Pi}'\right][\mathbf{\Omega}_t^{-1} \otimes \mathbf{W}_t'\mathbf{W}_t] = \text{Vec}\left[\mathbf{\Pi}'\right]\sum_{t=1}^{T}[\mathbf{\Omega}_t^{-1} \otimes \mathbf{W}_t'\mathbf{W}_t\right] \tag{19}$$

and eventually we obtain the final form:

$$\text{Vec}[\mathbf{\Pi}'] = \left(\sum_{t=1}^{T}[\mathbf{\Omega}_t^{-1} \otimes \mathbf{W}_t'\mathbf{W}_t]\right)^{-1}\text{Vec}\left[\sum_{t=1}^{T}\mathbf{W}_t'\mathbf{X}_t\mathbf{\Omega}_t^{-1}\right] \tag{20}$$

Equation 20 enables us to obtain, given any guess of $\{\gamma,\ \mathbf{\Omega}_E,\ \mathbf{\Omega}_R\}$, the associated $\mathbf{\Pi}$, and thus the likelihood: it will be sufficient to iterate over the guesses to find the global maximum. Since the problem presents itself as highly non-linear, use the Markov Chain Monte Carlo (MCMC) method developed by Chernozhukov and Hong (2003), implemented with the Metropolis-Hastings (MH) algorithm. The procedure consists in building a chain $\mathbf{\Psi} = \{\text{Chol}\,(\mathbf{\Omega}_C)\,,\ \text{Chol}\,(\mathbf{\Omega}_E)\}$ of drawings converging to the true distribution of parameters. We leave out $\gamma$, which is calibrated, and draw the Cholesky decomposition of the covariance matrices to ensure that $\mathbf{\Omega}_E$ and $\mathbf{\Omega}_R$ are always positive definite.
The MH algorithm is initialised with a $\mathbf{\Psi}^0$ entry, which is estimated from a linearised version of the model. A new *candidate* member $\mathbf{\Theta}$ will be generated as $\mathbf{\Theta} = \mathbf{\Psi}^0 + \psi$, with $\psi$ i.i.d. $\sim \mathcal{N}\left(\mathbf{0}, \mathbf{\Sigma}_\psi\right)$. $\mathbf{\Theta}$ is accepted and becomes $\mathbf{\Psi}^1$ if it improves the convergence of the chain, that is with probability $\min\left\{1,\ \exp\left[\mathcal{L}\left(\mathbf{\Theta}\right) - \mathcal{L}\left(\mathbf{\Psi}^0\right)\right]\right\}$.
$\mathbf{\Sigma}_\psi$ is adjusted on the fly to target an acceptance rate of around 30%. We perform 200.000 iterations and discard the first half as burn-in period.

# B Benchmark specification

Below are reported the main results for a benchmark specification where the model is estimated on the financial cycle described in Section 3.2.

## B.1 Impulse responses

Our focus is on the response of GDP to a fiscal shock via government expenditure. Together with the non-linear impulse responses presented above in Section 2.2, we also provide for comparison linear responses computed using the two extreme regime matrices identified by the model, $\mathbf{\Pi}_E$ and $\mathbf{\Pi}_R$ of Equation (1). This amounts to showing the IRFs of two distinct linear models with no interaction with each other.

A baseline and a debt augmented specification are considered for the set of $\mathbf{X}_t$ variables, namely $\mathbf{X}_t = [g_t,\ \tau_t,\ y_t,\ Pc_t]$ and $\mathbf{X}_t = [g_t,\ \tau_t,\ d_t,\ y_t,\ Pc_t]$, where $g$ denotes government expenditure; $\tau$ is tax revenues; $y$ represents GDP; $Pc$ is private credit (normalized by GDP); and $d$ denotes public debt (normalized by GDP). All variables are first differences of the log real series. We consider shocks of $\pm 1\%$ and $\pm 5\%$ to U.S. government expenditure, roughly corresponding to $\pm 0.15\%$ and $\pm 0.8\%$ of GDP. While the larger shock may look *too* large, the American Recovery and Reinvestment Act (ARRA) of 2009 (2009) stimulus package delivered an estimated combined impact of roughly 2.5% of GDP in the first year of enactment, as explained in The Congress of the United States - Congressional Budget Office (2012). Furthermore, the most recent debate on a grand stimulus package encourages us to be confident in using a relatively large shock.

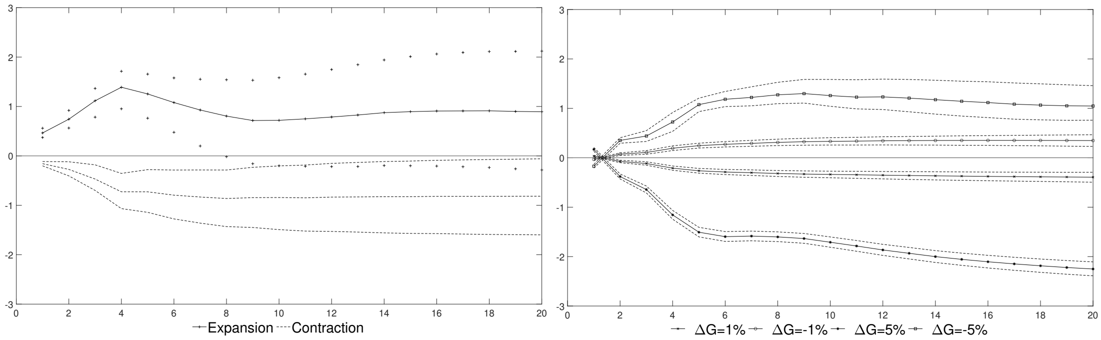
We also perform a scenario analysis considering two different environments in which the fiscal shock is delivered: a typical expansion and a typical recession. This complements the results presented in the main section, where the shock is timed to the most recent phase of the cycle. Our methodology involves building a *typical regime-specific* history of quarters and appending it to the history of realizations, effectively feeding the recursive mechanism of the model with an artificial set of values. This allows us to use the GIRF approach to investigate the effects of a fiscal shock imposed during a specific state of the economy and the cycle without sacrificing the smooth transitioning nature of our model. To build such a history, we use a discriminating criterion – the chronology published by the National Bureau of Economic Research of business cycle dates (as in Figure 2) – and we select every quarter in any given regime together with its lags. We then take the median value of the variables, thus obtaining a median representative recessionary or expansionary history.

To further explore the dynamics of fiscal shocks, GDP, and debt reactions of the augmented baseline, we also explicitly assume the relation between a government expenditure shock and the outstanding stock of public debt. We keep our methodology as simple and straightforward as possible and we impart a contemporaneous shock of the same size, and opposite sign, to both government expenditure and the stock of public debt, thus assuming that every expenditure increase is entirely financed via deficit spending and, at the same time, that a budget cut is only aimed to restructuring the stock of debt.

### B.1.1 Baseline specification

We start by presenting results for our baseline specification, including the main variables of government expenditure, tax revenues, and GDP, augmented by private credit, that is $\mathbf{X}_t = [g_t,\ \tau_t,\ y_t,\ Pc_t]$. We show results for both the full sample, up to the last quarter of 2019, and for a shorter sample not including the Great Recession. Figure 10 presents the GDP responses to a fiscal expenditure shock for the full sample.

Figure 10: Baseline specification, GDP reaction



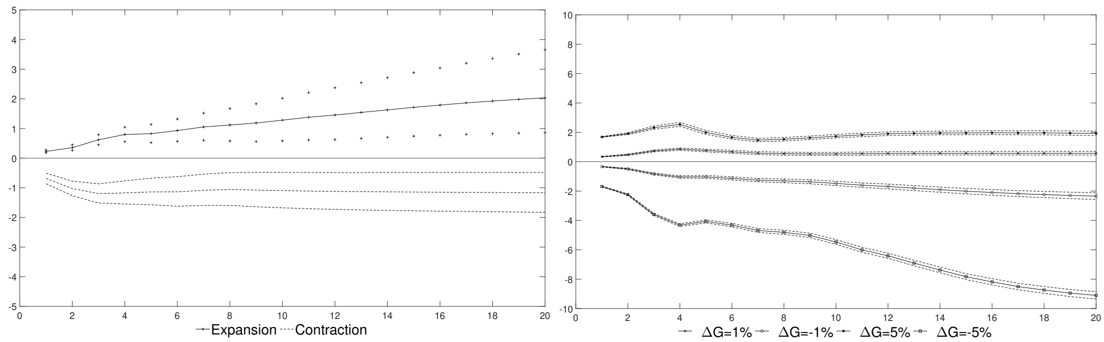

(a) Linear IRF (b) Generalised IRF

Note: Cumulative linear (a) and generalized (b) impulse responses. Percentage GDP response to a unit standard deviation (a) or to percentages of government expenditure (b) fiscal shock. $\Delta G$ denotes the variation in government expenditure, the percentage is the size of the shock. STVAR includes public expenditure, tax revenues, GDP, and private credit. Confidence bands are at 5th and 95th percentile.

The linear model clearly shows that the impact reaction to the same shock is stronger during an expansion period, rather than a contraction. The peak value reached is also higher when the economy is flourishing, despite happening during the same quarter for both the regimes, about a year after the shock. However, the long-run value is similar and in both cases it falls around the unity. The shock response also looks strongly pro-cyclical.

The generalized impulse responses present a number of interesting points. First, the impact and the long-run equilibrium value of the GDP reaction are opposite in sign, drawing a clear line between short, and medium and long-term equilibrium. Moreover, while the shocks are linearly scaled, the responses are not. Evidently there exists a phenomenon of diminishing returns to increasing shocks, where a larger negative shock has a limited, non-proportional, expansionary effect on the economy. The existence of such an asymmetric effect further justifies the choice of a non-linear model. Finally, negative expenditure shocks yield a positive GDP reaction and vice versa, seemingly endorsing austerity-like policies á la Alesina and Ardagna (2013). We impute to the presence of the Great Recession within the sample. Indeed this seems to be the case again, as results shown in Figure 11 suggest.

Figure 11: Shorter baseline (not including the Great Recession), GDP reaction



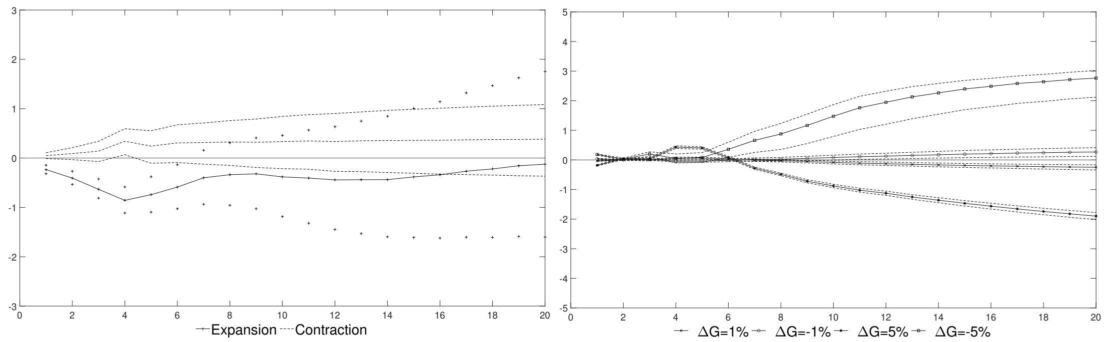

(a) Linear IRF (b) Generalised IRF

Note: Cumulative linear (a) and generalized (b) impulse responses. Percentage GDP response to a unit standard deviation (a) or to percentages of government expenditure (b) fiscal shock. $\Delta G$ denotes the variation in government expenditure, the percentage is the size of the shock. STVAR includes public expenditure, tax revenues, GDP, and private credit. Confidence bands are at 5th and 95th percentile.

Setting aside the differences due to a shorter sample (limited to the fourth quarter of 2008), the main divergence from the baseline is the effect of negative (positive) shocks being negative (positive) on the economy. A significant commonality, on the other hand, is the persistence of the diminishing returns of expansionary (in their effects) shocks, where there seems to be a limit to how much the economy is boostable via fiscal stimulus.

Figure 12 shows instead the response of the ratio of private credit-to-GDP, which we take as an indicator of the financial environment as already shown in Borio (2014). Two features are worth mentioning: the difference between a more dynamic short-run and a stabler long-run, and the marked difference between smaller and larger stimuli. Since the GDP reaction in Figure 10 looks smooth at every horizon, the overall conclusion we can draw is that the private credit reacts robustly pro-cyclically only after the short-period, pushing the ratio in the same direction as the GDP.

Figure 12: Baseline specification, credit-to-GDP response



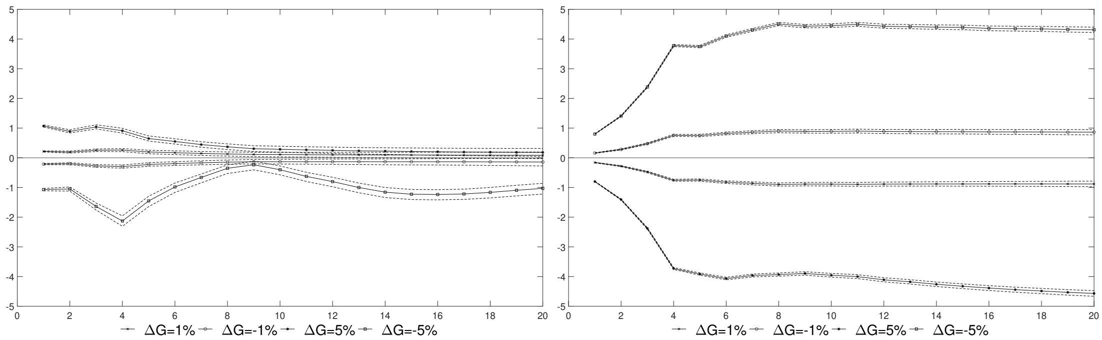

(a) Linear IRF (b) Generalised IRF

Note: Cumulative linear (a) and generalized (b) impulse responses. Percentage private credit response to a unit standard deviation (a) or to percentages of government expenditure (b) fiscal shock. $\Delta G$ denotes the variation in government expenditure, the percentage is the size of the shock. STVAR includes public expenditure, tax revenues, GDP, and private credit. Confidence bands are at 5th and 95th percentile.

### B.1.2 Scenario analysis, baseline specification

State-contingent IRFs computed in what we defined as *typical expansions* and *typical recessions*, and presented in Figure 13, feature a number of striking differences, aside from the general dynamic of the GDP response. In a typical expansion the fiscal stimulus is pro-cyclical and smaller in absolute value than in a typical recession. Moreover, the phenomenon of diminishing returns to larger shocks appears only during typical expansion, to the point where the long-term expansionary effect of a larger fiscal shock is almost identical to the response of the smaller one and very close to zero.

Figure 13: Baseline specification, scenario analysis



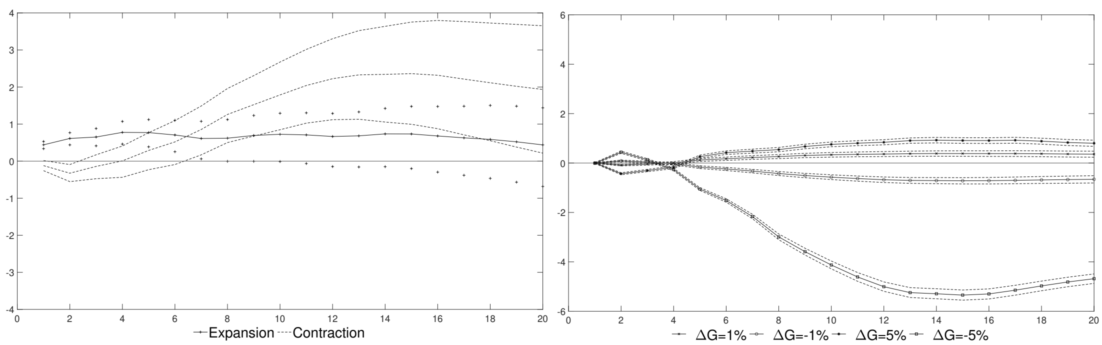

(a) Typical expansion (b) Typical recession

Note: Cumulative generalized impulse responses to a fiscal shock delivered in a median representative recession or expansion. $\Delta G$ denotes the variation in government expenditure, the percentage is the size of the shock. STVAR includes public expenditure, tax revenues, GDP, and private credit. Confidence bands are at 5th and 95th percentile.

### B.1.3 Augmented specification

Next we change the model specification to $\mathbf{X}_t = [g_t,\ \tau_t,\ d_t,\ y_t,\ Pc_t]$, augmenting the previous one with public debt. Rather than including it as it is, we choose to normalize it by GDP to obtain not a measure of the debt stock as such, but rather an indicator of fiscal burden relative to the size of the economy. Due to data availability our sample is now shorter, starting in 1966Q1. Figure 14 illustrates GDP reaction to a fiscal shock under the new specification.

Figure 14: Augmented specification, GDP response




(a) Linear IRF (b) Generalised IRF

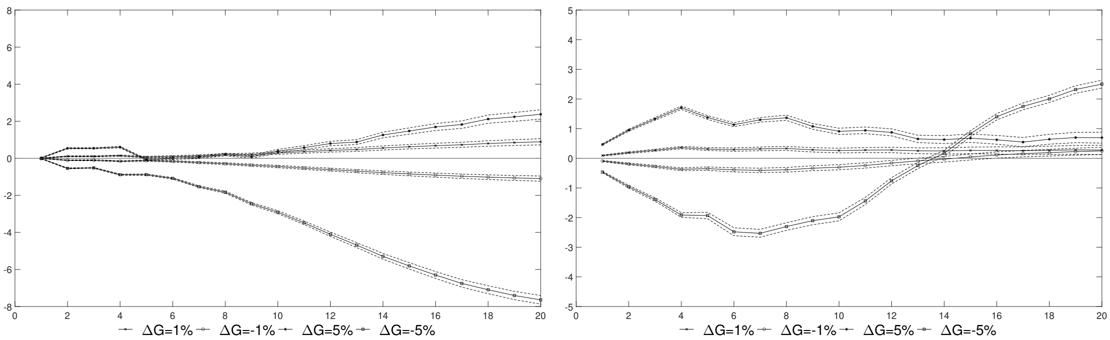
Note: Cumulative linear (a) and generalized (b) impulse responses. Percentage GDP response to a unit standard deviation (a) or to percentages of government expenditure (b) fiscal shock. $\Delta G$ denotes the variation in government expenditure, the percentage is the size of the shock. STVAR includes public expenditure, tax revenues, public debt, GDP, and private credit. Confidence bands are at 5th and 95th percentile.

Linear responses are now more diverse and less clearly identified, likely due to the lower number of observations used to estimate coefficients for one more variable. If the response during an extreme expansion appears rather stable, the contraction phase yields more dynamic behaviour, with a medium- and long-run response definitely larger than the expansionary counterpart.

The non-linear GIRFs keep some of the features shown by the baseline specification, such as the inversion in the sign of the responses between short and long periods, the impact effect being considerably smaller than the equilibrium long-run value, and the presence of diminishing returns of the expansionary response, where the larger shock does not yield a proportionally larger reaction. However, the most striking difference is in the concordance of the sign of shocks and reactions, where now a positive (negative) fiscal shock brings forth a positive (negative) GDP response. Such an effect appears to be entirely due to augmenting the specification with the ratio of public debt-to-GDP rather than due to the reduced sample size.

To complement our analysis, Figure 15 presents the evolution of the private credit-to-GDP and public debt-to-GDP following the fiscal shock.

Figure 15: Augmented specification, credit-to-GDP and debt-to-GDP response




(a) Generalised IRF (b) Generalised IRF

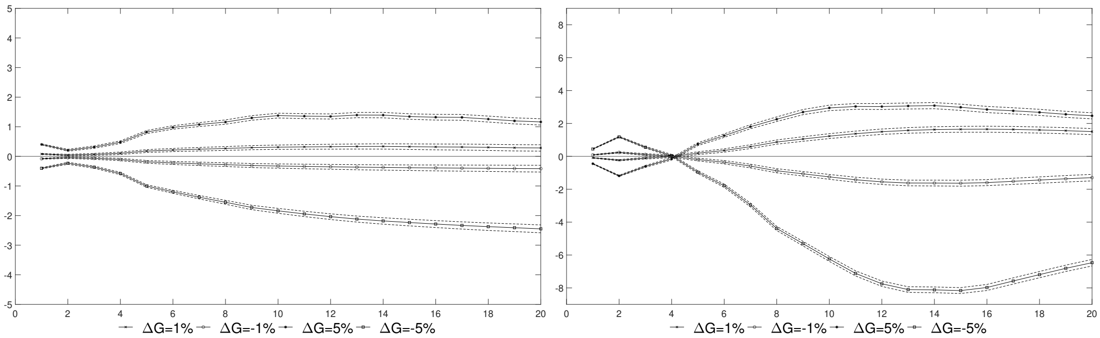
Note: Generalised impulse responses. Percentage private credit and debt response to a fiscal shock. $\Delta G$ denotes the variation in government expenditure, the percentage is the size of the shock. STVAR includes public expenditure, tax revenues, public debt, GDP and private credit. Confidence bands are at 5th and 95th percentile.

The credit behaviour is consistent with what we already observed for the baseline specification: results suggest that private credit moves strongly pro-cyclically, as a positive (negative) stimulus is paired with a growth (fall) of the private credit-to-GDP ratio. On the other hand, the behaviour of public debt-to-GDP ratio appears to be more diverse. Results clearly suggest that public debt variation will have the same sign as the fiscal shock. Rather than interpreting the result as public debt moving pro-cyclically, we favour the intuition that the fiscal shock itself is connected to the debt via deficit expansion of reduction. In this context, the long-run change in behaviour of the ratio after a larger negative shock can be seen as a first pro-cyclical moment, where the budget cut puts a downward pressure on the GDP, and it is directly used to lower the amount of public debt, followed by a phase where the debt dynamic wanes out (or even slightly rebounds), thus pushing up the ratio.

### B.1.4 Scenario analysis, augmented specification

Figure 16 presents the scenario analysis for our extended specification. Some key features of the general result of Figure 14 are carried over, such as the concordance between sign of the shock and sign of the response, the presence of a diminishing returns effect for the larger expansionary shock, and a discrepancy in the sign of the response between short- and long-run limited to the typical recession scenario. The most striking feature, however, is again that responses in a typical recession are larger than in a typical expansion.

Figure 16: Augmented specification, scenario analysis

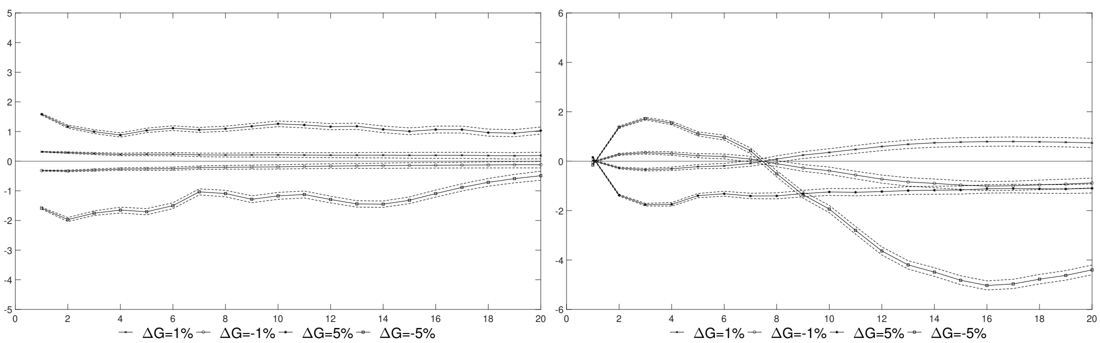



(a) Typical expansion (b) Typical recession

Note: Cumulative generalized impulse responses to a fiscal shock delivered in a median representative recession or expansion. $\Delta G$ denotes the variation in government expenditure, the percentage is the size of the shock. STVAR includes public expenditure, tax revenues, public debt, GDP, and private credit. Confidence bands are at 5th and 95th percentile.

A simple extension of the scenario analysis would be to explicitly assume how the fiscal shock influences the stock of public debt. We simulate an expenditure increase financed via public debt and a debt consolidation achieved via a one-time budget cut. Figure 17 shows that the results are qualitatively similar to the case of a typical expansion, with slightly higher magnitudes of responses in the short-run, and then lower in the long-term. On the other hand, the dynamics appear more diverse in a typical recession scenario. In the first place, we now have two inversions in the sign of the responses, one immediately after the impact and the other about two years in. However, such inversions do not affect the larger positive shock. Overall, the message yielded by the scenario is truly insightful: both large and small debt consolidations during a recession end up being recessionary, exactly as with the case of a large fiscal stimulus. The only strategy which appears successful in boosting the economy when the stimulus weighs entirely on debt is a small-sized fiscal package.

Figure 17: Augmented specification, scenario analysis with a shock to public debt




(a) Typical expansion (b) Typical recession

Note: Cumulative generalized impulse responses to a fiscal shock delivered in a median representative recession or expansion. The same shock is applied with opposite signs to government expenditure and public debt. $\Delta G$ denotes the variation in government expenditure, the percentage is the size of the shock. STVAR includes public expenditure, tax revenues, public debt, GDP, and private credit. Confidence bands are at 5th and 95th percentile.

## C List of variables

We remained faithful to the original naming system used by McCracken and Ng (2016) and detailed in the appendix of their paper. The column tcode denotes the following data transformations for a series $x$: (1) no transformation; (2) $\Delta x_t$; (3) $\Delta^2 x_t$; (4) $\log(x_t)$; (5) $\Delta \log(x_t)$; (6) $\Delta^2 \log(x_t)$; (7) $\Delta\left(\frac{x_t}{x_{t-1}}\right)$. The FRED column gives mnemonics in FRED followed by a short description. The comparable series in Global Insight, from which the data are taken, is given in the column GSI.

Table 2: Group 1: output and income

| | id | tcode | FRED | Description | GSI | GSI: description |
|---|---|---|---|---|---|---|
| 1 | 1 | 5 | RPI | Real Personal Income | M_14386177 | PI |
| 2 | 2 | 5 | W875RX1 | Real personal income ex transfer receipts | M_145256755 | PI less transfers |
| 3 | 6 | 5 | INDPRO | IP Index | M_116460980 | IP: total |
| 4 | 7 | 5 | IPFPNSS | IP: Final Products and Nonindustrial Supplies | M_116460981 | IP: products |
| 5 | 8 | 5 | IPFINAL | IP: Final Products (Market Group) | M_116461268 | IP: final prod |
| 6 | 9 | 5 | IPCONGD | IP: Consumer Goods | M_116460982 | IP: cons gds |
| 7 | 10 | 5 | IPDCONGD | IP: Durable Consumer Goods | M_116460983 | IP: cons dble |
| 8 | 11 | 5 | IPNCONGD | IP: Nondurable Consumer Goods | M_116460988 | IP: cons nondble |
| 9 | 12 | 5 | IPBUSEQ | IP: Business Equipment | M_116460995 | IP: bus eqpt |
| 10 | 13 | 5 | IPMAT | IP: Materials | M_116461002 | IP: matls |
| 11 | 14 | 5 | IPDMAT | IP: Durable Materials | M_116461004 | IP: dble matls |
| 12 | 15 | 5 | IPNMAT | IP: Nondurable Materials | M_116461008 | IP: nondble matls |
| 13 | 16 | 5 | IPMANSICS | IP: Manufacturing (SIC) | M_116461013 | IP: mfg |
| 14 | 17 | 5 | IPB51222s | IP: Residential Utilities | M_116461276 | IP: res util |
| 15 | 18 | 5 | IPFUELS | IP: Fuels | M_116461275 | IP: fuels |
| 16 | 19 | 1 | NAPMPI | ISM Manufacturing: Production Index | M_110157212 | NAPM prodn |
| 17 | 20 | 2 | CUMFNS | Capacity Utilization: Manufacturing | M_116461602 | Cap util |

Table 3: Group 2: labour market

| | id | tcode | FRED | Description | GSI | GSI: description |
|---|---|---|---|---|---|---|
| 1 | 21* | 2 | HWI | Help-Wanted Index for United States | | Help wanted indx |
| 2 | 22* | 2 | HWIURATIO | Ratio of Help Wanted/No. Unemployed | M_110156531 | Help wanted/unemp |
| 3 | 23 | 5 | CLF16OV | Civilian Labor Force | M_110156467 | Emp CPS total |
| 4 | 24 | 5 | CE16OV | Civilian Employment | M_110156498 | Emp CPS nonag |
| 5 | 25 | 2 | UNRATE | Civilian Unemployment Rate | M_110156541 | U: all |
| 6 | 26 | 2 | UEMPMEAN | Average Duration of Unemployment (Weeks) | M_110156528 | U: mean duration |
| 7 | 27 | 5 | UEMPLT5 | Civilians Unemployed - Less Than 5 Weeks | M_110156527 | U <5 wks |
| 8 | 28 | 5 | UEMP5TO14 | Civilians Unemployed for 5–14 Weeks | M_110156523 | U 5-14 wks |
| 9 | 29 | 5 | UEMP15OV | Civilians Unemployed - 15 Weeks & Over | M_110156524 | U 15+ wks |
| 10 | 30 | 5 | UEMP15T26 | Civilians Unemployed for 15–26 Weeks | M_110156525 | U 15-26 wks |
| 11 | 31 | 5 | UEMP27OV | Civilians Unemployed for 27 Weeks and Over | M_110156526 | U 27+ wks |
| 12 | 32* | 5 | CLAIMSx | Initial Claims | M_15186204 | UI claims |
| 13 | 33 | 5 | PAYEMS | All Employees: Total nonfarm | M_123109146 | Emp: total |
| 14 | 34 | 5 | USGOOD | All Employees: Goods-Producing Industries | M_123109172 | Emp: gds prod |
| 15 | 35 | 5 | CES1021000001 | All Employees: Mining and Logging: Mining | M_123109244 | Emp: mining |
| 16 | 36 | 5 | USCONS | All Employees: Construction | M_123109331 | Emp: const |
| 17 | 37 | 5 | MANEMP | All Employees: Manufacturing | M_123109542 | Emp: mfg |
| 18 | 38 | 5 | DMANEMP | All Employees: Durable goods | M_123109573 | Emp: dble gds |
| 19 | 39 | 5 | NDMANEMP | All Employees: Nondurable goods | M_123110741 | Emp: nondbles |
| 20 | 40 | 5 | SRVPRD | All Employees: Service-Providing Industries | M_123109193 | Emp: services |
| 21 | 41 | 5 | USTPU | All Employees: Trade, Transportation & Utilities | M_123111543 | Emp: TTU |
| 22 | 42 | 5 | USWTRADE | All Employees: Wholesale Trade | M_123111563 | Emp: wholesale |
| 23 | 43 | 5 | USTRADE | All Employees: Retail Trade | M_123111867 | Emp: retail |
| 24 | 44 | 5 | USFIRE | All Employees: Financial Activities | M_123112777 | Emp: FIRE |
| 25 | 45 | 5 | USGOVT | All Employees: Government | M_123114411 | Emp: Govt |
| 26 | 46 | 1 | CES0600000007 | Avg Weekly Hours: Goods-Producing | M_140687274 | Avg hrs |
| 27 | 47 | 2 | AWOTMAN | Avg Weekly Overtime Hours: Manufacturing | M_123109554 | Overtime: mfg |
| 28 | 48 | 1 | AWHMAN | Avg Weekly Hours: Manufacturing | M_14386098 | Avg hrs: mfg |
| 29 | 49 | 1 | NAPMEI | ISM Manufacturing: Employment Index | M_110157206 | NAPM empl |
| 30 | 127 | 6 | CES0600000008 | Avg Hourly Earnings: Goods-Producing | M_123109182 | AHE: goods |
| 31 | 128 | 6 | CES2000000008 | Avg Hourly Earnings: Construction | M_123109341 | AHE: const |
| 32 | 129 | 6 | CES3000000008 | Avg Hourly Earnings: Manufacturing | M_123109552 | AHE: mfg |

Table 4: Group 3: housing

| | id | tcode | FRED | Description | GSI | GSI: description |
|---|---|---|---|---|---|---|
| 1 | 50 | 4 | HOUST | Housing Starts: Total New Privately Owned | M_110155536 | Starts: nonfarm |
| 2 | 51 | 4 | HOUSTNE | Housing Starts, Northeast | M_110155538 | Starts: NE |
| 3 | 52 | 4 | HOUSTMW | Housing Starts, Midwest | M_110155537 | Starts: MW |
| 4 | 53 | 4 | HOUSTS | Housing Starts, South | M_110155543 | Starts: South |
| 5 | 54 | 4 | HOUSTW | Housing Starts, West | M_110155544 | Starts: West |
| 6 | 55 | 4 | PERMIT | New Private Housing Permits (SAAR) | M_110155532 | BP: total |
| 7 | 56 | 4 | PERMITNE | New Private Housing Permits, Northeast (SAAR) | M_110155531 | BP: NE |
| 8 | 57 | 4 | PERMITMW | New Private Housing Permits, Midwest (SAAR) | M_110155530 | BP: MW |
| 9 | 58 | 4 | PERMITS | New Private Housing Permits, South (SAAR) | M_110155533 | BP: South |
| 10 | 59 | 4 | PERMITW | New Private Housing Permits, West (SAAR) | M_110155534 | BP: West |

Table 5: Group 4: consumption, orders, and inventories

| | id | tcode | FRED | Description | GSI | GSI: description |
|---|---|---|---|---|---|---|
| 1 | 3 | 5 | DPCERA3M086SBEA | Real personal consumption expenditures | M_123008274 | Real Consumption |
| 2 | 4* | 5 | CMRMTSPLx | Real Manu. and Trade Industries Sales | M_110156998 | M&T sales |
| 3 | 5* | 5 | RETAILx | Retail and Food Services Sales | M_130439509 | Retail sales |
| 4 | 60 | 1 | NAPM | ISM: PMI Composite Index | M_110157208 | PMI |
| 5 | 61 | 1 | NAPMNOI | ISM: New Orders Index | M_110157210 | NAPM new ordrs |
| 6 | 62 | 1 | NAPMSDI | ISM: Supplier Deliveries Index | M_110157205 | NAPM vendor del |
| 7 | 63 | 1 | NAPMII | ISM: Inventories Index | M_110157211 | NAPM Invent |
| 8 | 64 | 5 | ACOGNO | New Orders for Consumer Goods | M_14385863 | Orders: cons gds |
| 9 | 65* | 5 | AMDMNOx | New Orders for Durable Goods | M_14386110 | Orders: dble gds |
| 10 | 66* | 5 | ANDENOx | New Orders for Nondefense Capital Goods | M_178554409 | Orders: cap gds |
| 11 | 67* | 5 | AMDMUOx | Unfilled Orders for Durable Goods | M_14385946 | Unf orders: dble |
| 12 | 68* | 5 | BUSINVx | Total Business Inventories | M_15192014 | M&T invent |
| 13 | 69* | 2 | ISRATIOx | Total Business: Inventories to Sales Ratio | M_15191529 | M&T invent/sales |
| 14 | 130* | 2 | UMCSENTx | Consumer Sentiment Index | hhsntn | Consumer expect |

Table 6: Group 5: money and credit

| | id | tcode | FRED | Description | GSI | GSI: description |
|---|---|---|---|---|---|---|
| 1 | 3 | 5 | DPCERA3M086SBEA | Real personal consumption expenditures | M_123008274 | Real Consumption |
| 2 | 4* | 5 | CMRMTSPLx | Real Manu. and Trade Industries Sales | M_110156998 | M&T sales |
| 3 | 5* | 5 | RETAILx | Retail and Food Services Sales | M_130439509 | Retail sales |
| 4 | 60 | 1 | NAPM | ISM: PMI Composite Index | M_110157208 | PMI |
| 5 | 61 | 1 | NAPMNOI | ISM: New Orders Index | M_110157210 | NAPM new ordrs |
| 6 | 62 | 1 | NAPMSDI | ISM: Supplier Deliveries Index | M_110157205 | NAPM vendor del |
| 7 | 63 | 1 | NAPMII | ISM: Inventories Index | M_110157211 | NAPM Invent |
| 8 | 64 | 5 | ACOGNO | New Orders for Consumer Goods | M_14385863 | Orders: cons gds |
| 9 | 65* | 5 | AMDMNOx | New Orders for Durable Goods | M_14386110 | Orders: dble gds |
| 10 | 66* | 5 | ANDENOx | New Orders for Nondefense Capital Goods | M_178554409 | Orders: cap gds |
| 11 | 67* | 5 | AMDMUOx | Unfilled Orders for Durable Goods | M_14385946 | Unf orders: dble |
| 12 | 68* | 5 | BUSINVx | Total Business Inventories | M_15192014 | M&T invent |
| 13 | 69* | 2 | ISRATIOx | Total Business: Inventories to Sales Ratio | M_15191529 | M&T invent/sales |
| 14 | 130* | 2 | UMCSENTx | Consumer Sentiment Index | hhsntn | Consumer expect |

Table 7: Group 6: interest and exchange rates

| | id | tcode | FRED | Description | GSI | GSI: description |
|---|---|---|---|---|---|---|
| 1 | 84 | 2 | FEDFUNDS | Effective Federal Funds Rate | M_110155157 | Fed Funds |
| 2 | 85* | 2 | CP3Mx | 3-Month AA Financial Commercial Paper Rate | CPF3M | Comm paper |
| 3 | 86 | 2 | TB3MS | 3-Month Treasury Bill | M_110155165 | 3 mo T-bill |
| 4 | 87 | 2 | TB6MS | 6-Month Treasury Bill | M_110155166 | 6 mo T-bill |
| 5 | 88 | 2 | GS1 | 1-Year Treasury Rate | M_110155168 | 1 yr T-bond |
| 6 | 89 | 2 | GS5 | 5-Year Treasury Rate | M_110155174 | 5 yr T-bond |
| 7 | 90 | 2 | GS10 | 10-Year Treasury Rate | M_110155169 | 10 yr T-bond |
| 8 | 91 | 2 | AAA | Moody's Seasoned Aaa Corporate Bond Yield | | Aaa bond |
| 9 | 92 | 2 | BAA | Moody's Seasoned Baa Corporate Bond Yield | | Baa bond |
| 10 | 93* | 1 | COMPAPFFx | 3-Month Commercial Paper Minus FEDFUNDS | | CP-FF spread |
| 11 | 94 | 1 | TB3SMFFM | 3-Month Treasury C Minus FEDFUNDS | | 3 mo-FF spread |
| 12 | 95 | 1 | TB6SMFFM | 6-Month Treasury C Minus FEDFUNDS | | 6 mo-FF spread |
| 13 | 96 | 1 | T1YFFM | 1-Year Treasury C Minus FEDFUNDS | | 1 yr-FF spread |
| 14 | 97 | 1 | T5YFFM | 5-Year Treasury C Minus FEDFUNDS | | 5 yr-FF spread |
| 15 | 98 | 1 | T10YFFM | 10-Year Treasury C Minus FEDFUNDS | | 10 yr-FF spread |
| 16 | 99 | 1 | AAAFFM | Moody's Aaa Corporate Bond Minus FEDFUNDS | | Aaa-FF spread |
| 17 | 100 | 1 | BAAFFM | Moody's Baa Corporate Bond Minus FEDFUNDS | | Baa-FF spread |
| 18 | 101 | 5 | TWEXMMTH | Trade Weighted U.S. Dollar Index: Major Currencies | | Ex rate: avg |
| 19 | 102* | 5 | EXSZUSx | Switzerland/U.S. Foreign Exchange Rate | M_110154768 | Ex rate: Switz |
| 20 | 103* | 5 | EXJPUSx | Japan/U.S. Foreign Exchange Rate | M_110154755 | Ex rate: Japan |
| 21 | 104* | 5 | EXUSUKx | U.S./U.K. Foreign Exchange Rate | M_110154772 | Ex rate: UK |
| 22 | 105* | 5 | EXCAUSx | Canada/U.S. Foreign Exchange Rate | M_110154744 | EX rate: Canada |

Table 8: Group 7: prices

| | id | tcode | FRED | Description | GSI | GSI: description |
|---|---|---|---|---|---|---|
| 1 | 106 | 6 | PPIFGS | PPI: Finished Goods | M110157517 | PPI: fin gds |
| 2 | 107 | 6 | PPIFCG | PPI: Finished Consumer Goods | M110157508 | PPI: cons gds |
| 3 | 108 | 6 | PPIITM | PPI: Intermediate Materials | M_110157527 | PPI: int matls |
| 4 | 109 | 6 | PPICRM | PPI: Crude Materials | M_110157500 | PPI: crude matls |
| 5 | 110* | 6 | OILPRICEx | Crude Oil, spliced WTI and Cushing | M_110157273 | Spot market price |
| 6 | 111 | 6 | PPICMM | PPI: Metals and metal products: | M_110157335 | PPI: nonferrous |
| 7 | 112 | 1 | NAPMPRI | ISM Manufacturing: Prices Index | M_110157204 | NAPM com price |
| 8 | 113 | 6 | CPIAUCSL | CPI: All Items | M_110157323 | CPI-U: all |
| 9 | 114 | 6 | CPIAPPSL | CPI: Apparel | M_110157299 | CPI-U: apparel |
| 10 | 115 | 6 | CPITRNSL | CPI: Transportation | M_110157302 | CPI-U: transp |
| 11 | 116 | 6 | CPIMEDSL | CPI: Medical Care | M_110157304 | CPI-U: medical |
| 12 | 117 | 6 | CUSR0000SAC | CPI: Commodities | M_110157314 | CPI-U: comm. |
| 13 | 118 | 6 | CUUR0000SAD | CPI: Durables | M_110157315 | CPI-U: dbles |
| 14 | 119 | 6 | CUSR0000SAS | CPI: Services | M_110157325 | CPI-U: services |
| 15 | 120 | 6 | CPIULFSL | CPI: All Items Less Food | M_110157328 | CPI-U: ex food |
| 16 | 121 | 6 | CUUR0000SA0L2 | CPI: All items less shelter | M_110157329 | CPI-U: ex shelter |
| 17 | 122 | 6 | CUSR0000SA0L5 | CPI: All items less medical care | M_110157330 | CPI-U: ex med |
| 18 | 123 | 6 | PCEPI | Personal Cons. Expend.: Chain Index | gmdc | PCE defl |
| 19 | 124 | 6 | DDURRG3M086SBEA | Personal Cons. Exp: Durable goods | gmdcd | PCE defl: dlbes |
| 20 | 125 | 6 | DNDGRG3M086SBEA | Personal Cons. Exp: Nondurable goods | gmdcn | PCE defl: nondble |
| 21 | 126 | 6 | DSERRG3M086SBEA | Personal Cons. Exp: Services | gmdcs | PCE defl: service |

Table 9: Group 8: stock market

| | id | tcode | FRED | Description | GSI | GSI: description |
|---|---|---|---|---|---|---|
| 1 | 80* | 5 | S&P 500 | S&P's Common Stock Price Index: Composite | M_110155044 | S&P 500 |
| 2 | 81* | 5 | S&P: indust | S&P's Common Stock Price Index: Industrials | M_110155047 | S&P: indust |
| 3 | 82* | 2 | S&P div yield | S&P's Composite Common Stock: Dividend Yield | | S&P div yield |
| 4 | 83* | 5 | S&P PE ratio | S&P's Composite Common Stock: Price-Earnings Ratio | | S&P PE ratio |

# D Alternative factor analysis

We presents the results of the factor analysis performed on the whole sample available, from 1959M01 to 2020M10 in Table 10. The information criterion $PC_{p2}$ selects eight relevant factors, which collectively explain a fraction of 0.5055 of the panel variance.

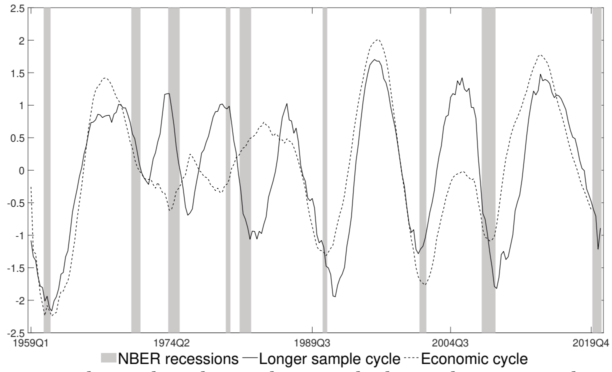
Table 10: Estimated factors and heavy loading series - $R^2(8) = 0.5055$

| $mR^2(1)$ | 0.1725 | $mR^2(2)$ | 0.0745 | $mR^2(3)$ | 0.0680 | $mR^2(4)$ | 0.0537 |
|---|---|---|---|---|---|---|---|
| ipmansics | 0.8055 | cusr0000sac | 0.4023 | aaaffm | 0.3690 | gs1 | 0.5058 |
| payems | 0.7989 | cusr0000sa0l2 | 0.3871 | t10yffm | 0.3568 | gs5 | 0.4989 |
| ipfpnss | 0.7579 | dndgrg3m086sbea | 0.3780 | baaffm | 0.3518 | aaa | 0.4772 |
| indpro | 0.7524 | cpitrnsl | 0.3733 | dndgrg3m086sbea | 0.3439 | tb6ms | 0.4594 |
| cumfns | 0.7409 | cpiaucsl | 0.3596 | cusr0000sac | 0.3437 | gs10 | 0.4592 |
| usgood | 0.7375 | pcepi | 0.3499 | cusr0000sa0l2 | 0.3252 | baa | 0.4238 |
| ipfinal | 0.6921 | cusr0000sa0l5 | 0.3444 | cpiaucsl | 0.3220 | cp3mx | 0.3690 |
| manemp | 0.6886 | cpiulfsl | 0.3128 | t5yffm | 0.3156 | tb3ms | 0.3665 |
| dmanemp | 0.6437 | wpsid61 | 0.2886 | cusr0000sa0l5 | 0.3024 | twexafegsmthx | 0.1824 |
| ipbuseq | 0.6290 | wpsfd49502 | 0.2816 | pcepi | 0.2868 | houst | 0.1819 |
| $mR^2(5)$ | 0.0480 | $mR^2(6)$ | 0.0339 | $mR^2(7)$ | 0.0298 | $mR^2(8)$ | 0.0252 |
| t1yffm | 0.5144 | s&p pe ratio | 0.3447 | s&p 500 | 0.3206 | twexafegsmthx | 0.3828 |
| tb6smffm | 0.4914 | s&p 500 | 0.2844 | s&p: indust | 0.3162 | exszusx | 0.1868 |
| tb3smffm | 0.4445 | s&p: indust | 0.2832 | vxoclsx | 0.2513 | conspi | 0.1825 |
| t5yffm | 0.4174 | s&p div yield | 0.2645 | uemp15ov | 0.2384 | exusukx | 0.1652 |
| t10yffm | 0.3497 | awhman | 0.2002 | ces0600000007 | 0.2108 | ces3000000008 | 0.1538 |
| aaaffm | 0.2689 | ces0600000007 | 0.1950 | awhman | 0.2101 | exjpusx | 0.1293 |
| compapffx | 0.2589 | uemp15ov | 0.1460 | s&p div yield | 0.2091 | ces0600000008 | 0.1070 |
| baaffm | 0.2015 | umcsentx | 0.1454 | uemp27ov | 0.1458 | ustrade | 0.0932 |
| permit | 0.1789 | mzmsl | 0.1397 | s&p pe ratio | 0.0942 | acogno | 0.0805 |
| permitw | 0.1567 | m2sl | 0.1084 | uemp15t26 | 0.0883 | ustpu | 0.0795 |

Note: Note: Eight factors selected by the $PC_{p2}$ criterion and the ten series loading the most on each factor. The table also reports the total variation explained by the eight factors ($R^2(8)$), the additional variation explained by adding the $k$th factor ($mR^2(k)$). As an example, the eight factors explain together 50.55% of the panel variation, while $mR^2(1) = 0.1725$ is the quota explained solely by the first factor. Moreover, 0.8055 is the fraction of variation of the series ipmansics explained by the first factor.

Factor interpretation is compatible with the results from the shorter sample. Factors 1 to 5 still carry information, respectively, on real production, prices, forward looking variables, interest rate, and a mixture of forward looking and housing variables. Factors 6 and 7 have explanatory power for the stock market and the labour market sectors, while the last factor concentrates on exchange rates. Figure 18 shows the cycle estimated on the longer sample contrasted with the index we used in our analysis.

Figure 18: Alternative economic cycle estimated on 8 factors

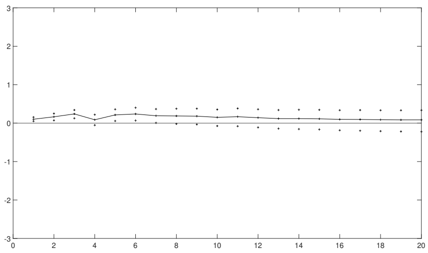



Note: Contrasting the economic cycle used in the analysis with the cycle estimated using a sample up to 2020M10. The selection criteria picks 8 significant factors.

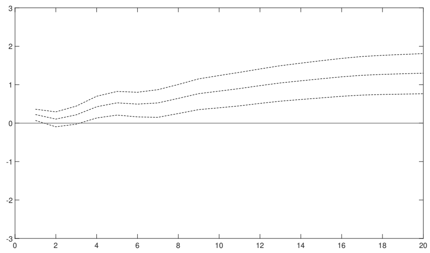
# E Additional figures for baseline specification

Our baseline specification includes (log real) government expenditure, tax revenues, GDP, and it is augmented with an estimate of the cycle.

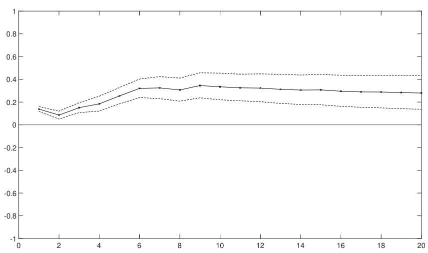
Figure 19: Linear and generalised IRFs, GDP response



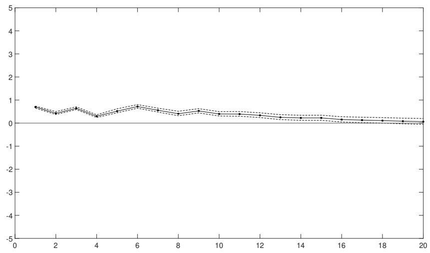
(a) Linear expansion



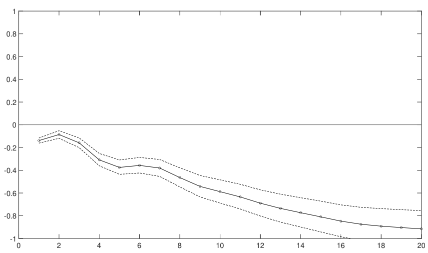
(b) Linear contraction



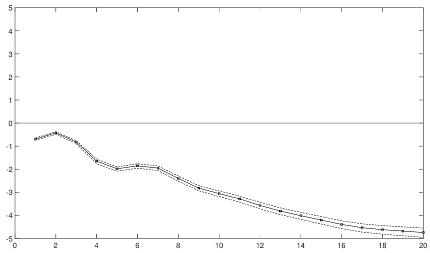
(c) +1% shock



(d) +5% shock



(e) −1% shock

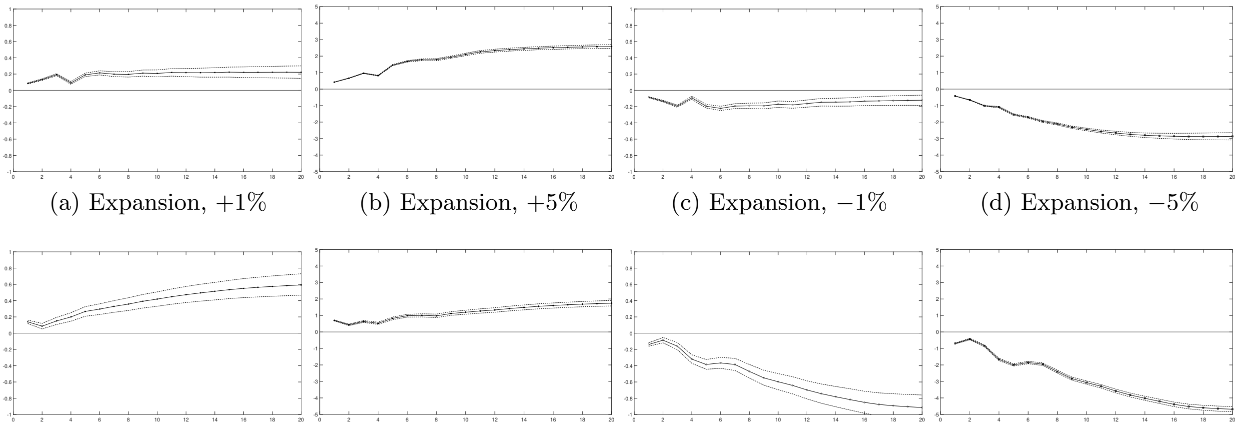


(f) −5% shock

Note: Cumulative linear (a, b) and generalised impulse responses. Percentage GDP response to a unit standard deviation (a, b) or to percentages of fiscal shock. STVAR includes public expenditure, tax revenues, GDP, and private credit. Confidence bands are at 5th and 95th percentile.

## E.1 Scenario analysis

Figure 20: Scenario analysis, GIRFs for typical scenarios



(a) Expansion, +1% (b) Expansion, +5% (c) Expansion, −1% (d) Expansion, −5%

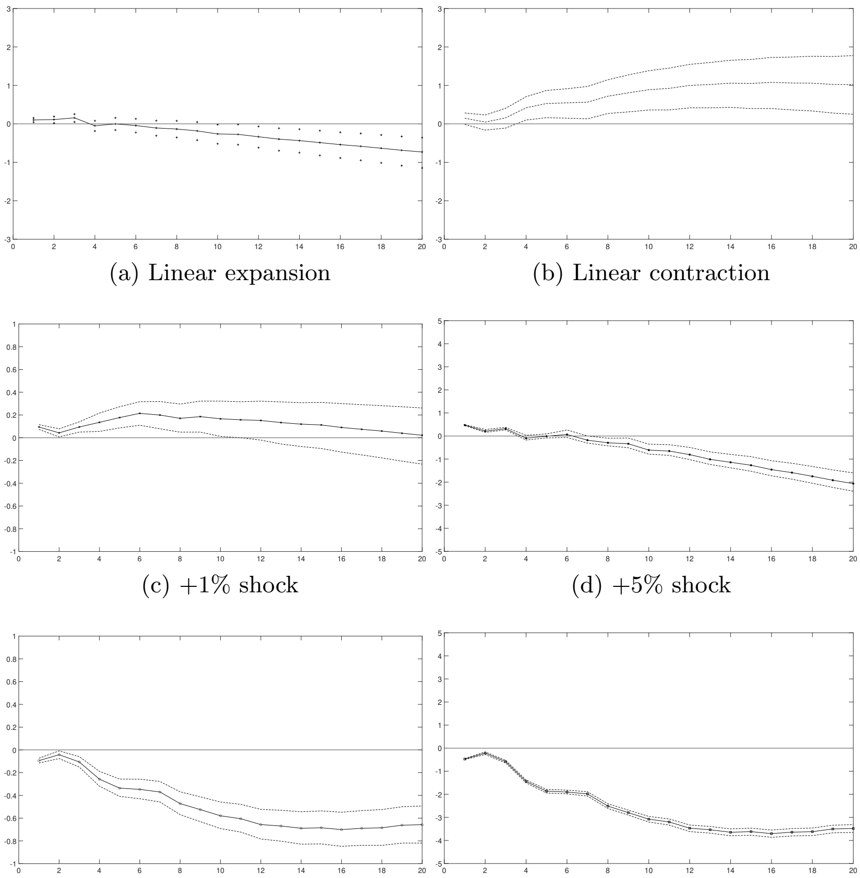
(e) Recession, +1% (f) Recession, +5% (g) Recession, −1% (h) Recession, −5%

Note: Cumulative generalised impulse responses to a shock triggered in a median representative recession or expansion. STVAR includes public expenditure, tax revenues, GDP, and private credit. Confidence bands are at 5th and 95th percentile.

# F Additional figures for augmented specifications

We present additional figures for our two augmented specifications, one extending the baseline with a measure of private credit, and the other with public debt. Both the additional variables are normalized by GDP to carry information on financial stress and fiscal space, rather than on the variables themselves.

Figure 21: Linear and generalised IRFs, GDP response



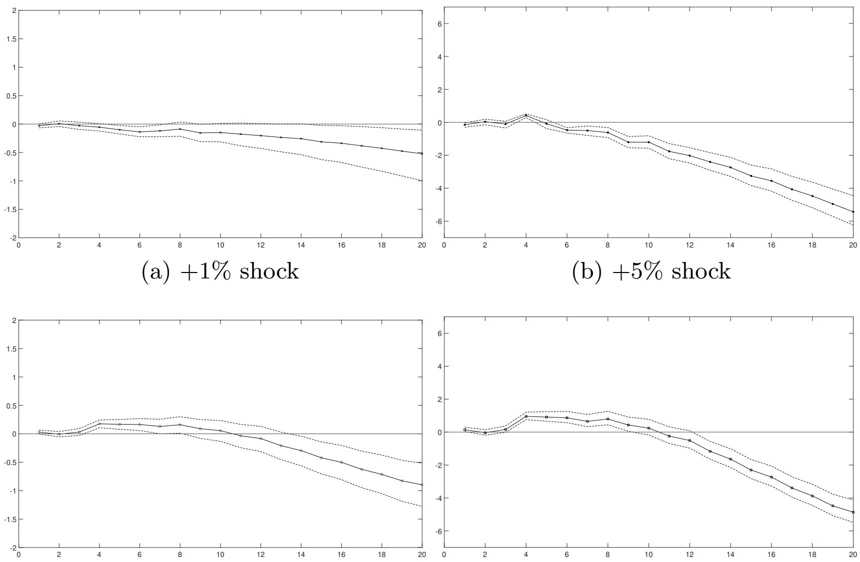
(a) Linear expansion (b) Linear contraction

(c) +1% shock (d) +5% shock

(e) −1% shock (f) −5% shock

Note: Cumulative linear (a, b) and generalised impulse responses. Percentage GDP response to a unit standard deviation (a, b) or to percentages of fiscal shock. STVAR includes public expenditure, tax revenues, GDP, and private credit. Confidence bands are at 5th and 95th percentile.

Figure 22: Generalised IRFs, credit-to-GDP response



(a) +1% shock

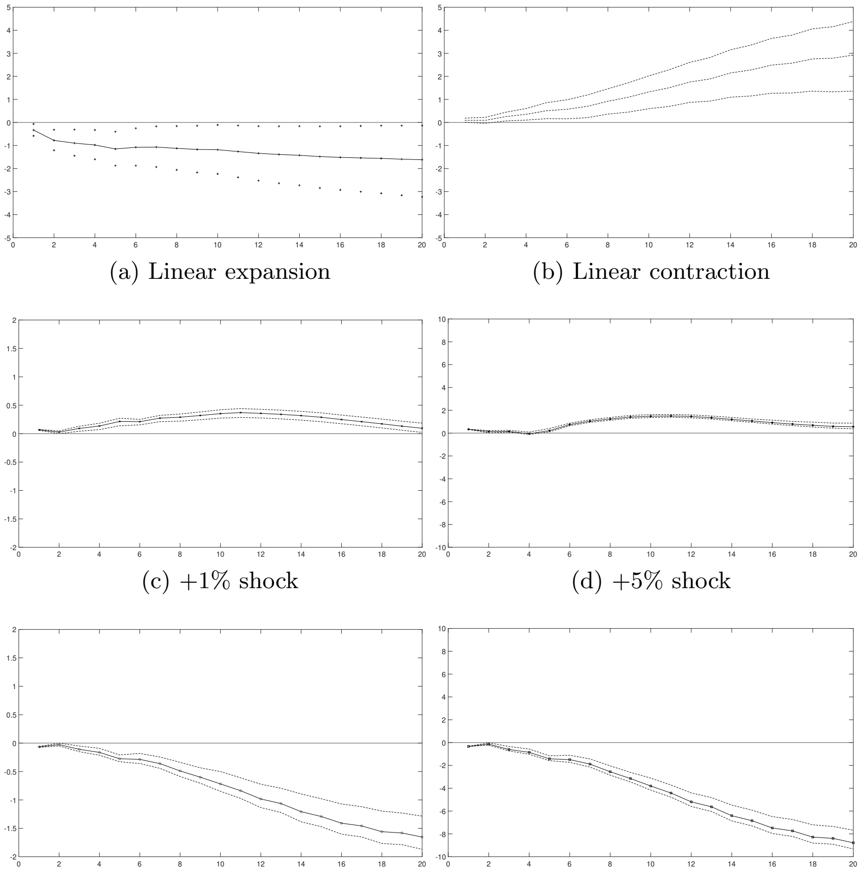
(b) +5% shock

(c) −1% shock

(d) −5% shock

Note: Generalised impulse responses. Percentage private credit response to percentages of fiscal shock. STVAR includes public expenditure, tax revenues, GDP, and private credit. Confidence bands are at 5th and 95th percentile.

Figure 23: Linear and generalised IRFs, GDP response



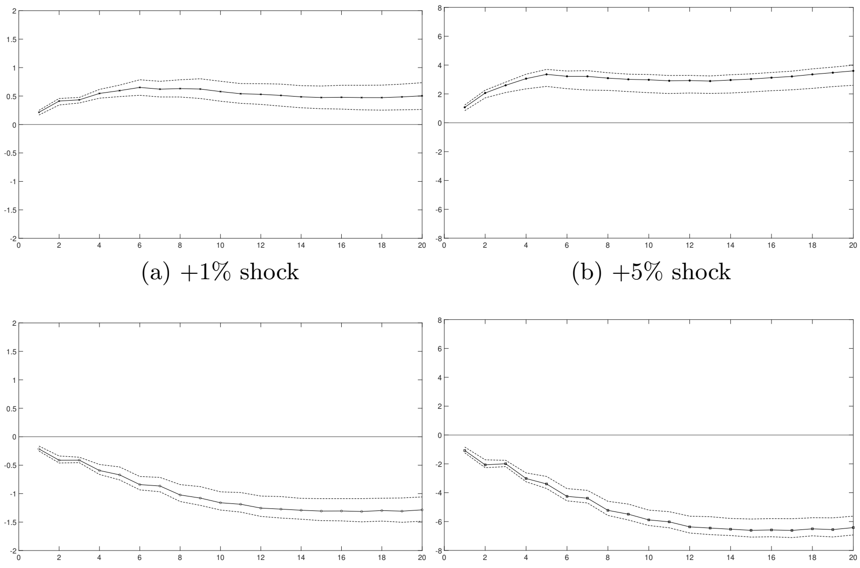
(a) Linear expansion (b) Linear contraction

(c) +1% shock (d) +5% shock

(e) −1% shock (f) −5% shock

Note: Cumulative linear (a, b) and generalised impulse responses. Percentage GDP response to a unit standard deviation (a, b) or to percentages of fiscal shock. STVAR includes public expenditure, tax revenues, public debt, and GDP. Confidence bands are at 5th and 95th percentile.

Figure 24: Generalised IRFs, debt-to-GDP response



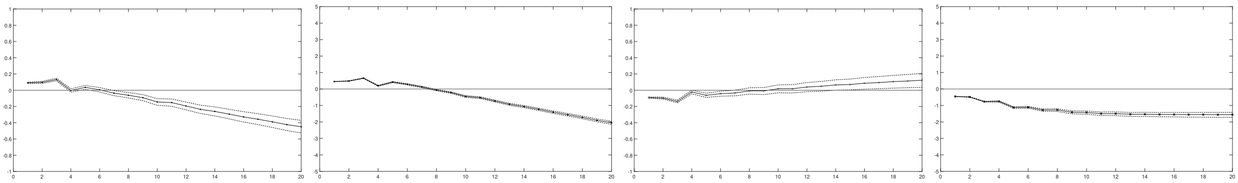
(a) +1% shock (b) +5% shock

(c) −1% shock (d) −5% shock

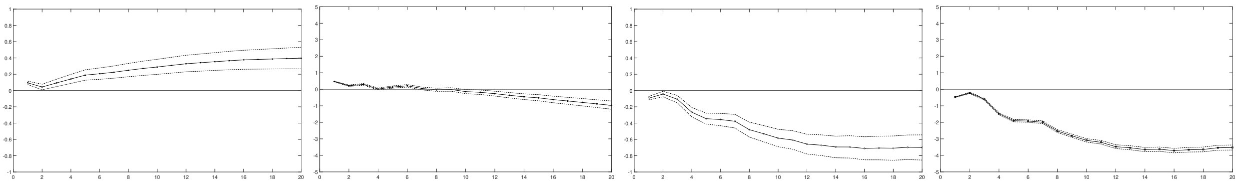
Note: Cumulative generalised impulse responses. Percentage public debt response to percentages of fiscal shock. STVAR includes public expenditure, tax revenues, public debt, and GDP. Confidence bands are at 5th and 95th percentile.

## F.1 Scenario analysis

Figure 25: Credit-augmented specification, scenario analysis, GIRFs for typical scenarios



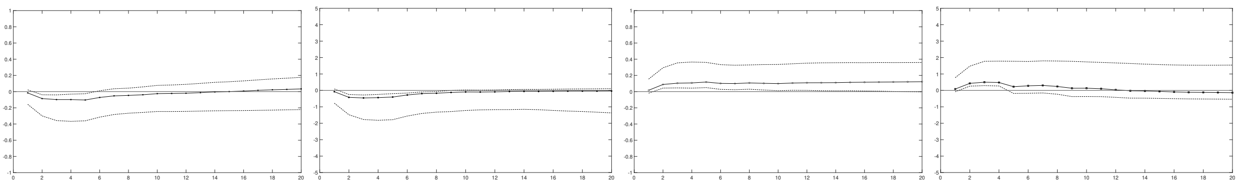
(a) Expansion, +1% (b) Expansion, +5% (c) Expansion, −1% (d) Expansion, −5%



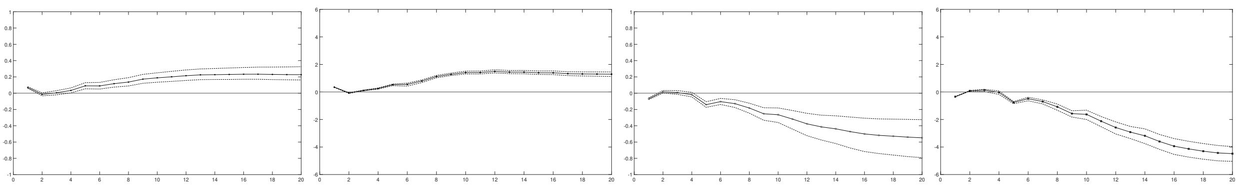
(e) Recession, +1% (f) Recession, +5% (g) Recession, −1% (h) Recession, −5%

Note: Cumulative generalised impulse responses to a shock triggered in a median representative recession or expansion. STVAR includes public expenditure, tax revenues, GDP, and private credit. Confidence bands are at 5th and 95th percentile.

Figure 26: Debt-augmented specification, scenario analysis, GIRFs for typical scenarios



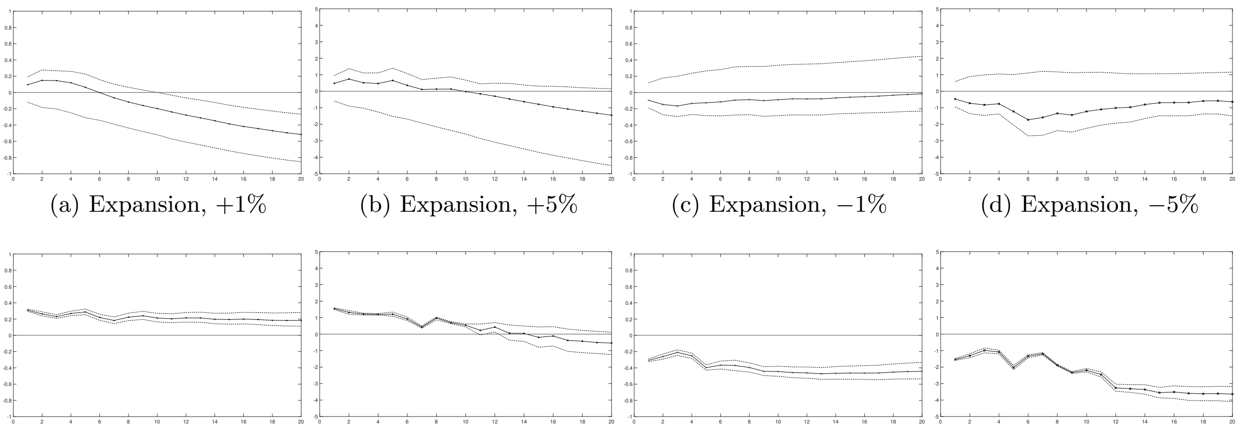
(a) Expansion, +1% (b) Expansion, +5% (c) Expansion, −1% (d) Expansion, −5%



(e) Recession, +1% (f) Recession, +5% (g) Recession, −1% (h) Recession, −5%

Note: Cumulative generalised impulse responses to a shock triggered in a median representative recession or expansion. STVAR includes public expenditure, tax revenues, public debt, and GDP. Confidence bands are at 5th and 95th percentile.

Figure 27: Debt-augmented specification, scenario analysis, GIRFs for typical scenarios with a shock to public debt



(a) Expansion, +1% (b) Expansion, +5% (c) Expansion, −1% (d) Expansion, −5%

(e) Recession, +1% (f) Recession, +5% (g) Recession, −1% (h) Recession, −5%

Note: Cumulative generalised impulse responses to a shock triggered in a median representative recession or expansion. The same shock is applied with opposite signs to government expenditure and public debt. STVAR includes public expenditure, tax revenues, public debt, and GDP. Confidence bands are at 5th and 95th percentile.